NCSM Great Tasks for Mathematics

Engaging Activities for

Effective Instruction and Assessment that Integrate

the Content and Practices of the

Common Core State Standards for Mathematics

A Resource from the
National Council of Supervisors of Mathematics

SAMPLE TASK

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Foreword

Coincidentally, the committee charged with compiling this set of tasks first met just a few weeks after the release of the PISA results for which Shanghai and Finland captured the attention of the world with their students' impressive performance.

Reflecting on student performance is what drove our initial discussions. We asked ourselves: How can a society improve the mathematical proficiency of its students? Our answer kept coming back to one simple measure — the degree to which the students we educate are able to take on the challenges of the society they inherit. That is, how will mathematical skills and knowledge help make the world a better place?

This publication is dedicated to providing a set of great tasks that mathematics educators can use to engineer learning experiences for our students that guide them into becoming the problem solvers of the future. We believe that great tasks allow students to become inventors. Great tasks allow students to demonstrate their thinking and give their teachers insight into student thinking. Great tasks provide opportunities to evaluate reasoning and examine sense making. In the end, instilling in students the "joy of figuring things out" will help mathematics teachers create the next generation empowered to use the amazing resources of mathematics to create a better world for all.

Introduction

Common Core State Standards
The United States has taken a clear direction toward building and implementing a set of core standards for all its students in mathematics education grades K–12. These new standards establish a set of high expectations for mathematical learning for all students. They also set high expectations for teaching, including standards for mathematical practices that will help parents, students, and educators achieve those goals.

> *Those interested in reading the entire set of Common Core State Standards for Mathematics can find a copy at:*
> *www.mathedleadership.org/docs/resources/ccss/CCSSI_Math%20Standards%20Expanded.pdf*

> *High School content by course "Pathways" can be found at:*
> *http://www.mathedleadership.org/docs/resources/ccss/CCSSI_Mathematics_Appendix_A--%20Model%20Course%20Pathways.pdf*

As important as the content standards are, it is the Standards for Mathematical Practice that address thinking and acting like a mathematician and that call for students to:

- **Make sense of problems and persevere in solving them.**
- **Reason abstractly and quantitatively.**
- **Construct viable arguments and critique the reasoning of others.**
- **Model with mathematics.**
- **Use appropriate tools strategically.**
- **Look for and make use of structure.**
- **Look for and express regularity in repeated reasoning.**

One way to help educators guide students in building proficiency at these practices is to model tasks and lessons like those presented in this publication.

What is a Great Task?
A great task:

- Revolves around an interesting problem – offering several methods of solution
- Is directed at essential mathematical content as specified in the standards
- Requires examination and perseverance – challenging students
- Begs for discussion – offering rich discourse on the mathematics involved
- Builds student understanding – following a clear set of learning expectations
- Warrants a summary look back – with reflection and extension opportunities

We know that students who innovate, create, discuss, engage, and are motivated to become problem solvers are more effective users of mathematics. Accordingly, mathematics instruction must focus on the development of these skills.

Presenting Great Tasks
"NCSM Great Task for Mathematics" follows the following design. Each task is presented with a set of Teacher Notes that provide an overview of the task, the Common Core State Standards Content and Practices standards that the task requires, prerequisite understandings, and specific suggestions for using the task. Each task includes 1) an Activity Launch that addresses key prerequisite understandings and assesses student readiness for the task, 2) the Core Task with which students are expected, individually and collaboratively, to wrestle, and 3) suggested Extension Activities to expand upon the learnings within the Core Task. In addition, a generic rubric is provided in this publication (see Figure 1) to help focus on the key aspects of completing the task.

Using Great Tasks

There are many ways we believe this collection can be used. Tasks can be incorporated into mathematics lessons as given or can be adapted to fit particular instructional needs. Tasks can also be used as assessments of student understanding of particular Common Core State Standards. These tasks can also be used as a part of professional development to help educators envision the types of instruction and assessment that are faithful to the vision of the new Common Core Standards.

Illustrating and Improving Students' Learning
The primary use of great tasks is as a tool for teachers to evaluate the depth of students understanding, to catch misconceptions, and as a foundation for classroom discourse designed to extend students' thinking.

Summative Assessment
After the mathematics content has been taught and students have the knowledge needed, these tasks can be used as a final group or individual summative assessment. After the review of the task, teachers will know what students understand and what concepts need to be revisited. These tasks will also provide information about students' ability to successfully meet the relevant Mathematical Practice Standards.

Assessment for Learning
When progressing through the material, tasks can be used as assignments along the way to measure students' progress and understanding. The information gathered can be used to guide teacher decisions about future instruction and provide feedback for students so they can improve their performance.

Professional Development for Teachers
Because the tasks in this collection were developed to illustrate both the philosophy and implementation needed to integrate great tasks in the curriculum, they can also serve as models for professional development. For example, presenting these tasks in a workshop can involve discussing the sample tasks and supporting teacher materials, and then developing similar sets of tasks. The exercise

of producing tasks is a nice way to help teachers jump into the essential content of the mathematics that is illustrated by these types of tasks.

Regardless of the audience or purpose, each of these tasks can be used as an individual, partner, small group or whole group task. Students and educators need the opportunity to participate in each of these types of activities.

- When used for **individual** work, a next step could be to take the time to allow students to work with a partner and evaluate each other's progress. After time spent with a partner the individual work could be revised and improved.

- The tasks could be completed as **partner** tasks so that they could discuss and talk about the information as they learn from one another and progress through the task.

- Using one of the tasks with a **small group** would be enhanced when students first read and think about any questions they have before joining the group. Groups can be facilitated by providing roles for each of the individuals. Each group can then share their solution with the entire class. Other groups should be encouraged to ask questions. The learning gained by taking the time to review student thinking helps students develop proficiency in the Standards for Mathematical Practices.

- With any of these methods, a **whole group** discussion of the task could provide cognitive closure when students share how they approached and completed the task. By listening and asking questions, students will learn different approaches to the problems from their peers' work. Planned questions will increase the value of the time spent reviewing the activity. Formative assessments can help improve student achievement by providing time to interact meaningfully and reflect on new information.

Providing Student Work
Another valuable tool to help students learn from a great task is to provide completed work and ask students to evaluate that work. They can look over what has been done to determine if the work is complete and in the process learn new approaches and begin to enhance their own skills for evaluating their own work.

Rubrics
Although rubrics can be used to evaluate student work and provide a numerical grade, they are just as important for providing specific feedback on critical elements of the task and the student work. Figure 1 provides a generic rubric with descriptions of four levels of accomplishment for each of four characteristics of the work.

Choose a rubric that focuses on skills, conceptual understandings and practices that you want to assess. You might choose to use additional rows if you wish to evaluate more than one skill, concept, or practice that a task requires. Although most of the tasks involve several different skills, concepts, and/or

practices, every skill, concept, or practice that students used does not need to be evaluated in the rubric. The rubric in this publication includes four performance levels; the fourth level reminds us that students at all levels need to be challenged and have opportunities for growth.

There are many ways to use a rubric. Often it is good when students can see how they will be evaluated before they begin the task. Another approach is to let them complete the task and then review the rubric and make changes before they submit their work. A rubric can be used without points attached by highlighting the appropriate sections. If points are a more appropriate use for your classroom, each section (row) can be allocated the appropriate number of points. For example, we could designate three rows for the task's mathematical skills, one for the mathematics concepts, two for mathematical practices and finally the last row for communication. The mathematical skills could be set up as 10 points for each row; the conceptual understanding as 20 points; practices could be 20 points each, and 10 points for communication. Then each column could be a percentage of the possible points; Needs improvement 0 to 50%, Approaching Proficiency 51% to 79%, Proficiency 80% to 95%, and Exemplary Distinction 96% to 100%. Each rubric must be tailored to what you want to learn about your students.

Figure 1: Generic Rubric for Assessing Student Work on Great Tasks

	Needs Improvement	**Approaches Proficiency**	**Demonstrates Proficiency**	**Exemplary Distinction**
Mathematics Skills	Little or no success with the mathematics skill. No workable solution is provided.	Part of the task is correct however gaps in skill and/or understanding are apparent.	Demonstrates solid execution of mathematical skill; presents a solution, which is correct and complete.	Work demonstrates rigorous mathematical skills and mastery that exceeds expectations.
Conceptual Understanding	Very little understanding of the mathematical concepts involved and/or misunderstood the task.	Some understanding of the relevant concepts is demonstrated.	Demonstrates knowledge of the mathematical concepts involved.	Work shows precise and thorough use of the mathematical concepts critical to successful completion of the task. Special insights or other exceptional qualities are included.
Mathematical Practice	Shows little or no progress toward demonstrating the mathematical practice.	Includes incomplete responses that demonstrate mathematics progress toward the mathematical practice.	Work demonstrates solid mathematical thinking and the ability to successfully use the mathematical practice.	Shows in-depth understanding of essential mathematical practice and eloquence or insight in the explanations of the practice.
Communication	Writing is confusing or absent.	There is some confusion in the writing and/or charts, diagrams. Mathematics is not clearly explained.	Addresses all processes and components of the task. Explanations are reasonable and clear to the audience.	Writes a comprehensive, compelling, and thoughtful solution. Diagrams are illuminating. Every component of the product is obvious to the audience.

Task Title: Bugs, Giraffes, Elephants and More

Grade Level: 4th

Task Overview:
The students will interpret line plots with scales written to the nearest quarter of a unit. The task requires students to calculate with fractions and leads to creating a foundation for basic statistics that will be assessed at the next grade level.

Prerequisite understandings:
Before interpreting line plots students need experience making line plots and for this activity they need to be able to use the quarter unit. Making and interpreting line plots are different skills and students will need both. This task is also connected to measurement skills and the ability to estimate a variety of measurements. Make sure students understand how to define "individuals" and values as they relate to the line plots.

CCSSM Content Standards:
Grade 4 Measurement & Data (4.MD)
Represent and Interpret Data
4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

CCSSM Mathematical Practices:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

Supplies Needed:
Rulers and yard sticks
Access to the internet or resources to research measurements

Teaching Notes:
    Launch activity:
    Make sure that the students have created accurate line plots. Review reading rulers to the nearest quarter inch. Collect some data from the students and make a line plot. Look at a number line divided in quarter lengths and make sure the students can read the lengths. Talk about each X representing a data value.

    Core task:
    Start by allowing the students to look at all the graphs and discuss ideas for the missing information in small groups. Next, pass out the activity sheets for the students to do with a partner or individually. Students should work with a partner to research and make decisions. Make sure they discuss their choices and are able to support the choices they make with facts. If they are having trouble making decisions they could use the research materials to help investigate measurements.

    Extension(s):
    Students could research the measure of an object that is measured to the nearest quarter of a unit and create their own line plots. They could then post their line plots around the room. The list of the topics researched would then be placed on the front board and students can be asked to match the topics to the plots. Specific interpretation questions could be asked as are used in the launch activity.

# Bugs, Giraffes, Elephants and More (Activity Launch)

These three activities review measurement and line plots to launch this task and assess student readiness.

Length of Our Pencils: Ask students to get out a ruler and measure the length of one of their pencils to the nearest quarter of an inch. Have the students select pencils that they have been using so there will be variability. Check the measurement with their partner and then put the measurement on the board where all the students will have access to it. After arranging the data from least to greatest, the group will create a line plot to the nearest quarter of an inch. Time should be spent discussing how it is created, what the data mean and how the plot can be interpreted. Possible questions to ask are: If we were to select two pencils and placed them end to end what are some of the lengths we could obtain? What is the shortest length? What is the longest? What length came up the most (mode)? What is the difference between the longest and shortest pencil (range)? Why do you think there are so many different lengths?

Objects in Our Class: Pair the students and ask them to select two items in the classroom that are less than one foot long. Each of the partners should measure the item to the nearest quarter of an inch. Once they have agreed on the length, they should put the name of the object and its length on the board. Partners should then work together to create a line plot of the class' data. Partners could then compare their plots with another pair of students. Each pair would then create questions for the other students in the class to answer based on the line plot. Spend time sharing and answering the questions.

Read my Line Plot: Show your students the Earthworm line plot. Ask students to write down the value of all of the numbers on the plot and then answer questions 1 to 6.

Length of My Earthworm in Inches
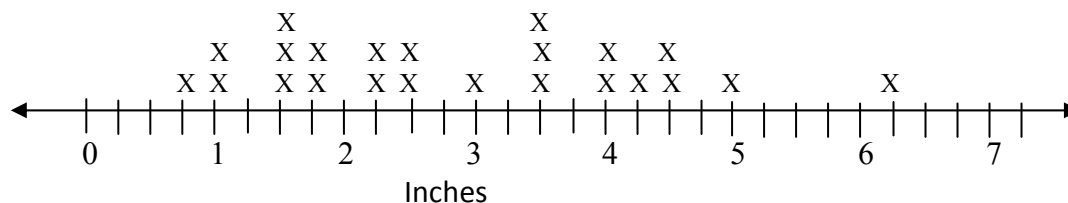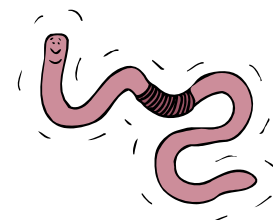
1. How many worms were measured?

2. List all the lengths of the worms.

3. What was the shortest worm? The longest worm?

4. What is the difference between the longest and shortest worm?

5. List 3 different lengths you could get if you were to put two worms end to end and measure. Show your work.

6. Why are the worm lengths so different?

Bugs, Giraffes, Elephants and More

Look at the five line plots below. The line plots contain data collected during a field trip to the zoo. Each plot is missing important information: what is being measured and the unit of measure being used.

Plot 1

Plot 2

Plot 3

```
              X           X
    X         X           X         X
    X         X X         X         X
    X         X X     X X     X X   X           X
    X X X X       X X X X X X     X     X X
  +--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+
  0        1         2         3         4
```
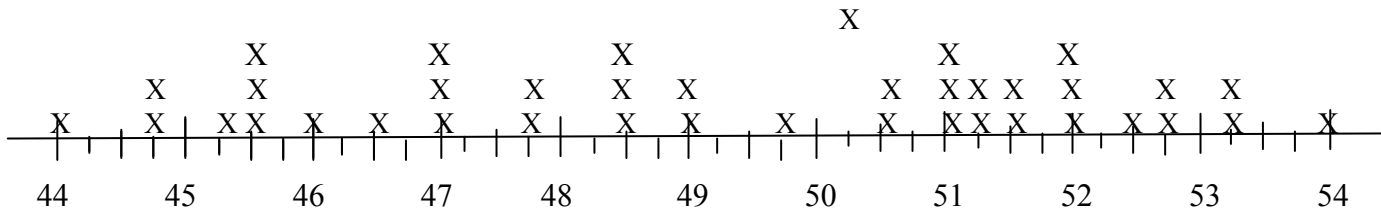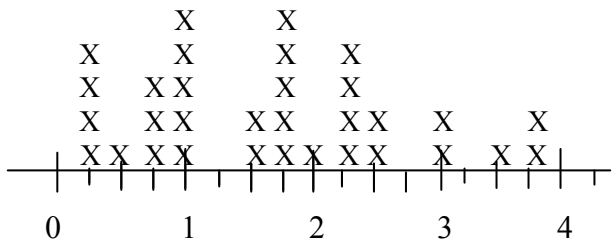
Plot 4

```
                          X
                          X
                X         X                 X
                X         X       X         X
          X     X   X     X X X   X   X X       X
  +--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+
  13         14        15        16        17        18        19
```

Plot 5

```
        X         X                         X
    X   X         X         X X       X     X                 X
    X       X X   X         X X   X   X     X X X   X   X   X     X X       X     X
  +--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+--+
  1         2         3         4         5         6         7         8         9         10        11
```
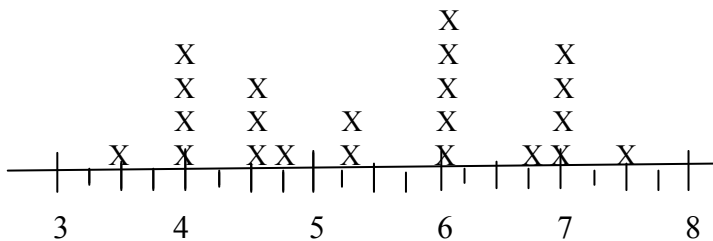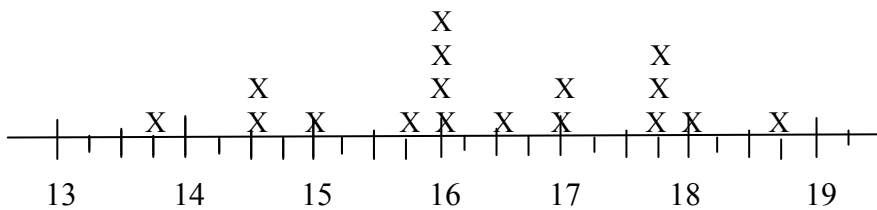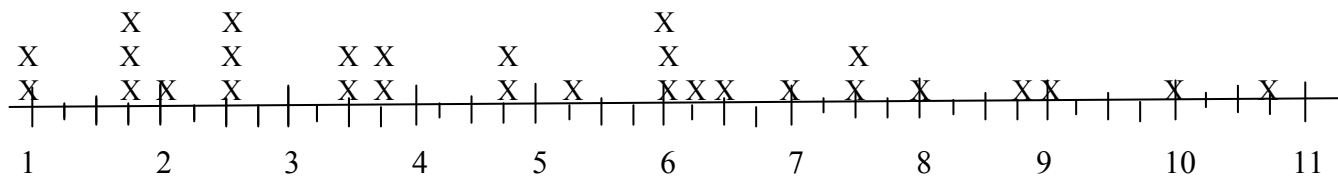
Bugs, Giraffes, Elephants and More

1. Which of the plots do you think shows:

    a. The length of bugs in cm collected on the last field trip?   _____

    b. The height of adult giraffes in feet?   _____

    c. The height of fourth graders in inches?   _____

    d. The weight in tons of adult elephants?   _____

2. Explain why you think the plot you picked for (c.) is the one that shows the heights of fourth graders.

3. Why do you think that other plots **don't** show the fourth graders' heights?

4. Which plot was not used? _____ What do you think this plot data could be about? Explain.

5. Look at plot 1.
    a. How many individuals are represented? _____

    b. What is the largest value?   _____

    c. What is the smallest value?   _____

    d. Find the difference between the largest and the smallest individual. _____ Explain what this tells you.

    e. What value has the most individuals? _____   How many? _____

6. Look at graph 4.
    a. How many individuals are represented? _____

    b. What is the largest value?   _____

    c. What is the smallest value?   _____

    d. Find the difference between the largest and the smallest individual. _____ Explain what this tells you.

    e. What value has the most individuals? _____   How many? _____

Results from the classroom:

Holly's work

1.   Which of the graphs do you think shows:

    a.   The length of bugs in cm. collected on the last field trip?   *graph 3*

    b.   The height of adult giraffes in feet?   *graph 4*

    c.   The height of fourth graders in inches?   *graph 2*

    d.   The weight in tons of adult elephants?   *graph 5*

2.   Explain why you think the graph you picked for c is the one that shows the heights of fourth graders.   *I picked graph 2 because there using inches but not feet.*

3.   Why do you think that other graphs **don't** show the fourth graders heights?   *The other graphs don't work because there eaithiver to high or to low*

4.   Which graph was not used? *graph 1*   What data do you think is shown? Explain.   *I thin think graph 1 because I think its cm.*

5.   Look at graph 4.
    a.   How many individuals are represented? *17*

    b.   What is the largest measured individual? *18.03*

    c.   What is the smallest? *13.03*

    d.   Find the difference between the largest and the smallest? *5 more whole feets*

    e.   What value has the most individuals? *16 feet* How many? *4*

6.   Look at graph 1.
    a.   How many individuals are represented? *22*

    b.   What is the largest measured individual? *7½*

    c.   What is the smallest? *3½*

    d.   Find the difference between the largest and the smallest? *4 less*

    e.   What value has the most individuals? *6 feet* How many ? *5*

Holly was comfortable with the first task of identifying the plots with the information provided. Although she had the same difficulty as most students, all of the students featured here selected plot 5 for part d when the correct answer should be plot 1. The data in plot 5 contains the possible elephant weights but it goes beyond feasible weights for mature elephants. She had more problems with the rest of the task. She refers to units in many of the answers on her own but does not know how to explain when asked in question 2. When asked what plot was not used, she could identify the plot but in her answer about the data she focused only on the unit and not the type of data. When she was reading the data she clearly struggled with the concept of quarter inches as in her answer for the largest data point in graph 4. Holly is trying to use some type of decimal system with an answer 18.03 rather than 18¾. She understands that it is three sections over but she does not know how to label it. When answering the other plot questions she had no trouble with the half unit measurements.

Khaled's work:

1. Which of the graphs do you think shows:

   a. The length of bugs in cm. collected on the last field trip? _3_

   b. The height of adult giraffes in feet? _4_

   c. The height of fourth graders in inches? _2_

   d. The weight in tons of adult elephants? _5_

2. Explain why you think the graph you picked for c is the one that shows the heights of fourth graders.

   I picked this graph because when you measure someone in inches when you finsh the 4th grader ends up being somewhere bewtwen 30 or 50 inches tall. I didnt pic the other ?

3. Why do you think that other graphs **don't** show the fourth graders heights?

   I don't think the other graphs worked because the numbers didn't go high enough for a fourth grader's height.

4. Which graph was not used? _1_ What data do you think is shown? Explain.

   The data I think is shown is snakes. I think this because snakes can be very long so I think 3 feet to 8 feet is a good estmate for a snake hight.

5. Look at graph 4.

   a. How many individuals are represented? _17_

   b. What is the largest measured individual? _18 3/4_

   c. What is the smallest? _13 3/4_

   d. Find the difference between the largest and the smallest? _5_

   e. What value has the most individuals? _16_ How many? _4_

6. Look at graph 1.

   a. How many individuals are represented? _22_

   b. What is the largest measured individual? _7 1/2_

   c. What is the smallest? _3 1/2_

   d. Find the difference between the largest and the smallest? _4_

   e. What value has the most individuals? _6_ How many ? _5_

Khaled has a more complete explanation for his plot selection and why none of the other plots will work. His answers are accurate and his selection of snakes for the remaining plot shows creativity and understanding of the question. Khaled has no problem with the rest of the task and has demonstrated his ability to work with line plots to the quarter unit.

Cassandra's work:

1. Which of the graphs do you think shows:
    a. The length of bugs in cm. collected on the last field trip? _3_
    b. The height of adult giraffes in feet? _4_
    c. The height of fourth graders in inches? _2_
    d. The weight in tons of adult elephants? _5_

2. Explain why you think the graph you picked for c is the one that shows the heights of fourth graders.
    I think graph 2 shows the heights of fourth graders because question c asks for heights in inches, not feet.

3. Why do you think that other graphs **don't** show the fourth graders heights?
    I think that the other graphs don't show the fourth grader's heights because the other graph's numbers are pretty low. The fourth graders are measuring in inches.

4. Which graph was not used? _1_ What data do you think is shown? Explain.
    I think it shows the height of gorillas in feet because all the graphs shows animals and humans so I thought the fourth graders took a trip to the zoo. Gorillas are usually human sized so I thought graph 1 would make sense.

5. Look at graph 4.
    a. How many individuals are represented? _17_
    b. What is the largest measured individual? $18\frac{3}{4}$
    c. What is the smallest? $13\frac{3}{4}$
    d. Find the difference between the largest and the smallest? _5_
    e. What value has the most individuals? _16_ How many? _4_

6. Look at graph 1.
    a. How many individuals are represented? _22_
    b. What is the largest measured individual? $7\frac{1}{2}$
    c. What is the smallest? $3\frac{1}{2}$
    d. Find the difference between the largest and the smallest? _4_
    e. What value has the most individuals? _6_ How many? _5_

Looking at Cassandra's work we see she was very interested in the setting and as well as having solid answers she demonstrates good reasoning skill when answering questions two through four.

David's work:

1. Which of the graphs do you think shows:

   a. The length of bugs in cm. collected on the last field trip? _____ 4

   b. The height of adult giraffes in feet? _____ 1

   c. The height of fourth graders in inches? _____ 3

   d. The weight in tons of adult elephants? _____ 5

2. Explain why you think the graph you picked for c is the one that shows the heights of fourth graders.

   I think 4 Because it is the smallist.

3. Why do you think that other graphs **don't** show the fourth graders heights?

   Because it dosint mack since if a long paper it tuns or 54 inches

4. Which graph was not used? _2_ What data do you think is shown? Explain.

   Because graph 2 Does not mack sence foreney of them.

5. Look at graph 4.
   a. How many individuals are represented? _13_

   b. What is the largest measured individual? _4_

   c. What is the smallest? _3_

   d. Find the difference between the largest and the smallest? _3_

   e. What value has the most individuals? _2_ How many? _54_

6. Look at graph 1.
   a. How many individuals are represented? _22_

   b. What is the largest measured individual? _34_

   c. What is the smallest? _17_

   d. Find the difference between the largest and the smallest? _4_

   e. What value has the most individuals? _2m_ How many? _54_

Finally we look at one of the struggling students, David. David struggles with all of the components of the task. His answers to the plot matching appear to be random. When students do not understand or have the skills to do the problem many are comfortable with any answer. David does not have an understanding of the size of the measurements. His bugs would be very large and while a student could successfully argue that plot 4 would work for bugs it would be difficult to support that the height of adult giraffes would range from 3½ to 7½ feet. His reasoning for the selection of plot 3 for the height of 4th graders is faulty and he may be confusing inches with feet but even that would be difficult to support a 1 and 2 foot high students. He says that Plot 2 does not make sense for any of the options. The rest of his answers show a complete inability to read the line plots and support interventions with this student.