



# CCGPS Frameworks Student Edition

## Mathematics

Fourth Grade Unit Two  
Fraction Equivalents



**Dr. John D. Barge, State School Superintendent**  
*"Making Education Work for All Georgians"*

## Unit 2 FRACTION EQUIVALENTS

### TABLE OF CONTENTS

Overview .....	3
Standards for Mathematical Content .....	3
Standards for Mathematical Practice .....	3
Enduring Understandings.....	5
Essential Questions .....	5
Concepts & Skills to Maintain.....	5
Selected Terms and Symbols.....	6
Strategies for Teaching and Learning.....	7
Evidence of Learning.....	7
Tasks .....	8
• Fraction Kits.....	9
• Their Fair Share .....	12
• Write About Fractions .....	16
• Benchmark Fractions .....	19
• More or Less .....	22
• Closest to 0, 1/2, or 1 .....	28
• Equivalent Fractions .....	31
• Making Fractions .....	38
• Factor Findings .....	41
• Finding Products.....	45
• Factor Trail Game.....	48
• The Sieve of Eratosthenes.....	51
Culminating Task	
Pattern Block Puzzles .....	56

## **OVERVIEW**

In this unit students will:

- identify visual and written representations of fractions
- understand representations of simple equivalent fractions
- understand the concept of mixed numbers with common denominators to 12
- convert mixed numbers to improper fractions and improper fractions to mixed fractions

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight **STANDARDS FOR MATHEMATICAL PRACTICE**: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, modeling mathematics, using appropriate tools strategically, attending to precision, looking for and making use of structure, and looking for and expressing regularity in repeated reasoning, should be addressed continually as well. The first unit should establish these routines, allowing students to gradually enhance their understanding of the concept of number and to develop computational proficiency.

To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the tasks listed under “Evidence of Learning” be reviewed early in the planning process. A variety of resources should be utilized to supplement. The tasks in these units illustrate the types of learning activities that should be utilized from a variety of sources.

## **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

**MCC4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

**MCC4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

## **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**\*\*\*Mathematical Practices 1 and 6 should be evident in EVERY lesson\*\*\***

## **ENDURING UNDERSTANDINGS**

- Mixed numbers and improper fractions can be used interchangeably.
- Fractions can be represented visually and in written form.
- Fractions with differing parts can be the same size.
- Fractions of the same whole can be compared.

## **ESSENTIAL QUESTIONS**

- How can common numerators or denominators be created?
- How can equivalent fractions be identified?
- How can fractions with different numerators and different denominators be compared?
- How can improper fractions and mixed numbers be used interchangeably?
- How do we apply our understanding of fractions in everyday life?
- What are some situations which factoring is appropriate
- What are the benefits of creating common denominators?
- What does it mean to factor?
- What is a fraction and how can it be represented?
- What is a mixed number and how can it be represented?
- What is an improper fraction and how can it be represented?
- What is factor?
- What is the relationship between a mixed number and an improper fraction?

## **CONCEPTS/SKILLS TO MAINTAIN**

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Identify and give multiple representations for the fractional parts of a whole (area model) or of a set, using halves, thirds, fourths, sixths, eighths, tenths and twelfths.
- Recognize and represent that the denominator determines the number of equally sized pieces that make up a whole.

- Recognize and represent that the numerator determines how many pieces of the whole are being referred to in the fraction.
- Compare fractions with denominators of 2, 3, 4, 6, 10, or 12 using concrete and pictorial models.

### **SELECTED TERMS AND SYMBOLS**

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Teachers should present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for elementary children. It has activities to help students more fully understand and retain new vocabulary. (i.e. The definition for *dice* actually generates rolls of the dice and gives students an opportunity to add them.)

<http://www.teachers.ash.org.au/jeather/maths/dictionary.html>

<http://intermath.coe.uga.edu/dictnary/>

**The terms below are for teacher reference only and are not to be memorized by the students.**

- common fraction
- composite
- denominator
- equivalent sets
- factor
- improper fraction
- increment
- mixed number
- numerator
- prime
- proper fraction
- term
- unit fraction
- whole number

### **STRATEGIES FOR TEACHING AND LEARNING**

- Students should be actively engaged by developing their own understanding.
- Mathematics should be represented in as many ways as possible by using graphs, tables, pictures, symbols, and words.
- Interdisciplinary and cross curricular strategies should be used to reinforce and extend the learning activities.
- Appropriate manipulatives and technology should be used to enhance student learning.

- Students should be given opportunities to revise their work based on teacher feedback, peer feedback, and metacognition which includes self-assessment and reflection.
- Students should write about the mathematical ideas and concepts they are learning.
- Books such as *Fraction Action* (2007) written and illustrated by Loreen Leedy and *Working with Fractions* (2007) by David A. Adler and illustrated by Edward Miller, are useful resources to have available for students to read during the instruction of these concepts.
- Consideration of all students should be made during the planning and instruction of this unit. Teachers need to consider the following:
  - What level of support do my struggling students need in order to be successful with this unit?
  - In what way can I deepen the understanding of those students who are competent in this unit?
  - What real life connections can I make that will help my students utilize the skills practiced in this unit?

### **EVIDENCE OF LEARNING**

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Represent and read proper fractions, improper fractions, and mixed numbers in multiple ways.
- Represent equivalent common fractions
- Use mixed numbers and improper fractions interchangeably.
- Compare fractions and express their relationships using the symbols,  $>$ ,  $<$ , or  $=$ .
- Interpret and represent a multiplication equation as a multiplicative comparison.
- Find and identify multiples and factors of a given whole number.
- Explore and analyze divisibility for common factors such as 2, 5, and 10.

## **TASKS**

The following tasks represent the level of depth, rigor, and complexity expected of all fourth grade students. These tasks or tasks of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. While some tasks are identified as performance tasks, they also may be used for teaching and learning.

<b>Scaffolding Task</b>	<b>Constructing Task</b>	<b>Practice Task</b>	<b>Performance Tasks</b>
Tasks that build up to the constructing task.	Constructing understanding through deep/rich contextualized problem solving tasks	Games/activities	Summative assessment for the unit

<b>Task Name</b>	<b>Task Type Grouping Strategy</b>	<b>Content Addressed</b>
Fraction Kits	Scaffolding Task <i>Individual Task</i>	Recognize equivalent fractions
Their Fair Share	Scaffolding Task <i>Individual Task</i>	Recognize equivalent fractions
Write About Fractions	Constructing Task <i>Small Group/Partner Task</i>	Write fraction number sentences, identify and represent equivalence
Benchmark Fractions	Constructing Task <i>Small Group/Partner Task</i>	Write fraction number sentences, identify and represent equivalence
More or Less	Practice Task <i>Small Group/Partner Task</i>	Write fraction number sentences, identify and represent equivalence
Closest to 0, 1/2, or 1	Practice Task <i>Small Group/Partner Task</i>	Write fraction number sentences, identify and represent equivalence
Equivalent Fractions	Practice Task <i>Individual/Partner Task</i>	Using multiplicative/finding equivalent fractions
Making Fractions	Practice Task <i>Partner Task</i>	Compare and order fractions
Factor Findings	Constructing Task <i>Partner Task</i>	Recognizing factors as prime and composite numbers
Finding Products	Constructing Task <i>Individual/Partner Task</i>	Understanding multiplicative identity with factors
Factor Trail Game	Practice Task <i>Individual/Partner Task</i>	Determining factor pairs
The Sieve of Eratosthenes	Practice Task <i>Individual/Partner Task</i>	Determining prime numbers less than 100
Pattern Block Puzzles	Performance/ <b>Culminating Task</b> <i>Individual/Partner Task</i>	Demonstrate an understanding of equivalent fractions

## **Scaffolding Task: Fraction Kits**

*Inspired by Math Solutions Publications Teaching Arithmetic: Lessons for Introducing Fractions by Marilyn Burns*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Constructing the idea that fractions are relationships, and that the size or amount of the whole matters, is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning from 2<sup>nd</sup> grade, with standards which refer to same-sized shares or equal shares. Students should have knowledge of vocabulary terms such as: *numerator and denominator*.

Some common misconceptions, found in *Math Misconceptions*, that children have include:

- Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes – circles, squares, and rectangles – they begin to think that all shapes are divided similarly.
- Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set

of 6 vehicles. The student may mistakenly confuse the set of cars as  $\frac{2}{4}$  of the unit instead of  $\frac{2}{6}$  or  $\frac{1}{3}$  (Bamberger, Oberdorf, & Schultz-Ferrell, 2010). Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

### **ESSENTIAL QUESTIONS**

- What is a fraction and how can it be represented?
- How can equivalent fractions be identified?

### **MATERIALS**

- Construction paper strips of equal length (8 different colors prepared prior to beginning task with students)

### **GROUPING**

Individual

### **TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

**Comments:** Be sure to cut the strips of construction paper accurately before asking students to make kits. Students should create their own kits since there are basic concepts about fractions to be learned from the process of creating the kit. It is important to discuss the concept of fair sharing. Ask them how they would share the strips of colored paper among various groups of people (two, three, four, six, eight, and sixteen). It is important that you allow time for students to determine the appropriate way to equally share the strips of paper among the various groups. For students who are unable to determine the appropriate measures, ensure they are exposed to strategies discovered by their peers through classroom discussions.

- To cut fourths, first have students cut halves, then fold and cut each half into fourths.
- To cut eighths, first have students cut fourths (as above), then cut each fourth in half to make eighths.
- To cut sixteenths, have students cut eighths (as above), then cut each eighth in half to make sixteenths.
- To cut thirds, use the concept of measurement. Tell students that the strip is 18 inches long. How many inches for each piece if we want thirds (shares with three people)? Students should have time for small group discussion and sharing.
- To cut sixths, follow the same idea as with thirds, but for sharing with six people. Again, students should have time for small group discussion and sharing.

This task is one that enables the construction of a manipulative that can be used with subsequent tasks. Therefore, it is important that all students are provided with an opportunity to construct their own fraction kit.

### **Task Directions:**

Cut 12-by-18-inch construction paper lengthwise into 3-by-18-inch strips. For the fractions 1 whole,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ , and  $\frac{1}{16}$ , you will need 5 different colors. To include  $\frac{1}{3}$ ,  $\frac{1}{6}$ , and  $\frac{1}{12}$ , you will need an additional 3 colors (for a total of 8 colors.)

- Ask students to take a strip of a particular color (that you choose/ or the class agrees upon), fold it in half, and cut it into two pieces. Have them label each piece  $\frac{1}{2}$  (and possibly also with the name ‘one half’) and discuss why this label is appropriate (because the pieces are the same size, each is one of the two pieces, which we represent as  $\frac{1}{2}$ ).
- Choose a color for a second strip and have the students fold and cut it into four equal pieces. Instruct students to label each piece  $\frac{1}{4}$  (and possibly with the name ‘one fourth’). Have students explain why the label is appropriate.
- Have students fold, cut, and label a third strip in eighths and a fourth strip in sixteenths. (For the sixteenths, students may need to fold a strip in half, cut it, and then fold each half into eighths.)
- Students leave one of the strips whole and label it 1 or  $\frac{1}{1}$  (ask students first what they should label it).
- For creating thirds, students would probably be inaccurate in folding. One strategy is to measure the strip, divide the length into three equal segments, and then measure and cut and label. For sixths, they can cut thirds and then fold the thirds in half. (The issue of why this works would be a good topic for class discussion.) Use a similar process for twelfths.

### **FORMATIVE ASSESSMENT QUESTIONS**

- What was the initial cut you made to your colored strips? Why?
- Are your labels appropriate? Why or why not? How do you know?
- Could you use the strips for halves or fourths as a template for making thirds or sixths? Why or why not?
- Which fraction piece is bigger,  $\frac{1}{2}$  or  $\frac{1}{4}$ ? Next,  $\frac{1}{3}$  or  $\frac{1}{6}$ ?

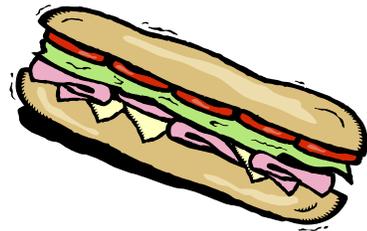
### **DIFFERENTIATION**

#### **Extension**

- Ask students to explain why making other fractions such as  $\frac{1}{7}$  would be difficult. Can you name other fractions that would be difficult? How would you go about making them?

#### **Intervention**

- Begin by having students work only with halves, thirds, and fourths only.
- Discuss how these pieces compare to the whole.



## **Scaffolding Task: Their Fair Share**

*Adapted from Fosnot, C. The Field Trip, Context for Learning Mathematics.*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(nxa)/(nxb)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Constructing the idea that fractions are relationships and that the size or amount of the whole matters is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning from 2<sup>nd</sup> grade, with standards which refer to same-sized shares or equal shares.

### **ESSENTIAL QUESTIONS**

- How can we use fair sharing to determine equivalent fractions?
- How do we know fractional parts are equivalent?

### **MATERIALS**

- Connecting cubes
- Equal length strips of paper
- Chart paper
- Fraction Kits from previous task (optional)

### **GROUPING**

small group or partner

## **TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

**Comments:** The numbers in this story have been chosen purposefully. Given the chosen numbers, students are catapulted into proportional reasoning. It is important that you facilitate discussions regarding their thinking at this point allowing them the opportunity to figure out how much each person in each group received.

When students begin to compare and/or add pieces together, some students may attempt to represent subs with the connecting cubes, but may not make equal size subs. If you see students using various sizes, be sure to point this out by asking if one group received bigger subs. Stay grounded in the context to help students realize the meaning of what they are doing. You want students to derive a common length sub in order to compare and/or add the fractional amounts. This idea is important for the construction of common denominators.

Make all materials available for use.

### **Task directions:**

Students will follow the directions below from the “Their Fair Share” student sheet.

A fourth- grade class traveled on a field trip in four separate vehicles. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

- The first group had 3 people and shared 2 subs equally.
- The second group had 4 people and shared 3 subs equally.
- The third group had 9 people and shared 6 subs equally.
- The last group had 6 people and shared 4 subs equally.

When they returned from the field trip, the children began to argue that the portion of the sandwiches they received was not fair, that some children got more to eat than others. Were they right? Or did everyone get the same amount?

Create a poster of the ideas and strategies you used to solve this problem. Be sure your poster is concise and clear presentations of the important ideas and strategies you want to present.

## **FORMATIVE ASSESSMENT QUESTIONS**

- What initial cut did you make to your subs? Why?
- Did you use all parts of the subs? Were each parts equivalent?
- (For students using connecting cubes) Were all subs the same size? What amount of cubes could you use to show all of the subs were the same size?

## **DIFFERENTIATION**

### **Extension**

- Encourage students to try a different strategy to determine if they will arrive at the same fractional parts.

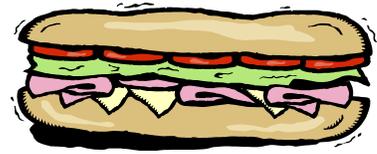
### **Intervention**

- Have students use strips of equal length paper to represent the submarine sandwiches.
- Encourage students to use connecting cubes. Have them begin with 12 cubes to get them thinking about how to show common length subs.

Name \_\_\_\_\_ Date \_\_\_\_\_

## Their Fair Share

### Directions



A fourth- grade class traveled on a field trip in four separate vehicles. The school provided a lunch of submarine sandwiches for each group. When they stopped for lunch, the subs were cut and shared as follows:

- The first group had 3 people and shared 2 subs equally.
- The second group had 4 people and shared 3 subs equally.
- The third group had 9 people and shared 6 subs equally.
- The last group had 6 people and shared 4 subs equally.

When they returned from the field trip, the children began to argue that the portion of the sandwiches they received was not fair, that some children got more to eat than others. Were they right? Or did everyone get the same amount?

Create a poster of the ideas and strategies you used to solve this problem. Be sure your poster is concise and clear presentations of the important ideas and strategies you want to present.

## **Constructing Task: Write About Fractions**

*Adapted from Fosnot, C. The Field Trip, Context for Learning Mathematics.*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(nxa)/(nxb)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Many students develop misconceptions about adding fractions, thinking the numerators can be added and denominators can be added. For example, they might think that

$$\frac{1}{3} + \frac{1}{2} \text{ equivalent to } \frac{2}{5}$$

### **ESSENTIAL QUESTIONS**

- How can you compare fraction quantities without adding fractions?
- How do you know fractions are equivalent?
- What can you do to decide whether your answer is reasonable?

### **MATERIALS**

- Connecting cubes
- Strips of equal length paper

- Write About Fractions task sheet
- Fraction Kits

## **GROUPING**

Individual

## **TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

**Comments:** students have had some opportunities to work with comparing unit fractions with different denominators in context. By exploring this procedure in context and examining the results, students are supported to develop deep understandings about fractions, thereby avoiding common misconceptions.

### **Task directions:**

Students will follow the directions below from the “Write About Fractions” task sheet.

Write a convincing argument for the following statement:

$$\frac{1}{2} + \frac{1}{3} \text{ does not equal } \frac{2}{5}$$

## **FORMATIVE ASSESSMENT QUESTIONS**

- What do you notice about the unit fractions you have created?
- How can drawing a model help you answer this question?
- What strategies can you use to help write about this topic?
- Can you give me an estimated size of the fraction you created with that cut?
- Does your answer make sense? How do you know?

## **DIFFERENTIATION**

### **Extension**

- In addition to this task, student can identify a fraction that is equal to  $\frac{1}{2} + \frac{1}{3}$ .

### **Intervention**

- Provide fraction strips the students can manipulate to determine their response.
- Encourage students to use words, numbers and/or pictures in their explanation.

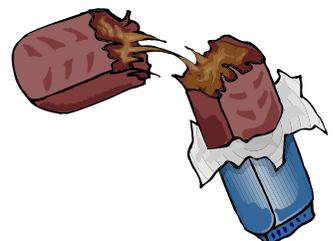
Name \_\_\_\_\_ Date \_\_\_\_\_

## Write About Fractions

Write a convincing argument for the following statement:

$$\frac{1}{2} + \frac{1}{3} \text{ does not equal } \frac{2}{5}$$

### Constructing Task: Benchmark Fractions



*Adapted from Fosnot, C. The Field Trip, Context for Learning Mathematics.*

## **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(nxa)/(nxb)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

## **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## **BACKGROUND KNOWLEDGE**

Students were familiarized with benchmark fractions in 2<sup>nd</sup> grade. They worked to partition circles and rectangles into halves and thirds. In 3<sup>rd</sup> grade students generated simple equivalent fractions and were required to explain why the fractions were equivalent.

## **ESSENTIAL QUESTIONS**

- How can benchmark fractions be used to compare fractions?

## **MATERIALS**

- Connecting cubes
- Strips of equal length paper
- Fraction Kits

## **GROUPING**

small group or partner

### **TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

**Comments:** For this task, if some students immediately say  $17/22$ , having constructed the idea of fractions as fair sharing division from previous tasks, acknowledge their thinking but explain you were wondering about how much that would be. In a real situation, no one is going to cut a candy bar into 22 pieces, and you are wondering about how much that amount is so that the fewest possible cuts could be made. Direct students to the guiding questions listed on their student sheet. The main focus of this task is on the development of reasonableness of judging the magnitude of fractional amounts.

#### **Task directions:**

Students will follow the directions below from the “Benchmark Fractions” task sheet.

Mrs. Toms’ 4<sup>th</sup> grade class just concluded a unit on the solar system. To celebrate how hard students worked, Mrs. Toms decided to purchase Milky Way candy bars for students to share. The grocery store only had 17 of the king size candy bars, but there are 22 students in her class. If Mrs. Toms buys the bars, about how much of a bar would each student receive? Is the amount about  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{3}{4}$ ,  $\frac{2}{3}$  or 1 whole? Where could one cut be made that would be a nice approximate?

### **FORMATIVE ASSESSMENT QUESTIONS**

- What do you notice about the unit fractions you have created?
- If you tried dividing, would you have the same results?
- Can you give me an estimated size of the fraction you created with that cut?
- Can you tell me what you are doing with the pieces of the bars as you work with them?
- Does your answer make sense? How do you know?

### **DIFFERENTIATION**

#### **Extension**

- Explain how the process for redistributing the pieces differs as the number of candy bars and/or number of students changes?

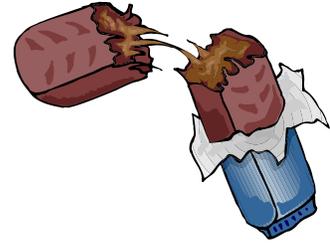
#### **Intervention.**

- Students can cut out their drawings of the bars then cut off pieces and move them to make approximate equivalent amounts.

Name \_\_\_\_\_ Date \_\_\_\_\_

## Benchmark Fractions

### Directions



Mrs. Toms' 4<sup>th</sup> grade class just concluded a science unit on the solar system. To celebrate how hard students worked, Mrs. Toms decided to purchase Milky Way candy bars for students to share. The grocery store only had 17 of the king size candy bars, but there are 22 students in her class. If Mrs. Toms buys the bars, about how much of a bar would each student receive? Is the amount about  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{3}{4}$ ,  $\frac{2}{3}$  or 1 whole? Where could one cut be made that would be a nice approximate?

### **Practice Task:** More or Less

*Adapted from Actions on Fractions, Navigating through Number and Operations in Grades3-5.*

## **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

## **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## **BACKGROUND KNOWLEDGE**

Students were familiarized with benchmark fractions in 2<sup>nd</sup> grade. They worked to partition circles and rectangles into halves and thirds. In 3<sup>rd</sup> grade students generated simple equivalent fractions and were required to explain why the fractions were equivalent.

## **ESSENTIAL QUESTIONS**

- What are benchmark fractions?
- How are benchmark fractions helpful when comparing fractions?

## **MATERIALS**

- More or Less game cards

## **GROUPING**

small group or partner

## **TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

**Comments:** Review the meaning of fraction and the ways in which the students can compare two fractions. Explore the patterns that the students have discovered for determining the larger or smaller fractions. Help students think about fractions in four groups: fractions with the same denominators, fractions with a numerator of 1, fractions with same numerators (other than 1), and fractions with different numerators and different denominators.

**Task directions:**

Students will follow the directions below from the “More or Less” student sheet.

Round 1: Benchmark fraction is  $\frac{3}{4}$

1. Pull a card from the top of the stack.
2. Determine if the fraction is larger or smaller than  $\frac{3}{4}$ . Explain to your partner(s) why you believe it is larger or smaller.
3. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
4. Pull another card and complete steps 2-3 until all cards have been used.

Round 2: Benchmark fraction  $\frac{1}{2}$

1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than  $\frac{1}{2}$ . Explain to your partner(s) why you believe it is larger or smaller.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 3: Benchmark fraction  $\frac{1}{3}$

1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than  $\frac{1}{3}$ . Explain to your partner(s) why you believe it is larger or smaller.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 4:

1. Pull two cards from the top of the stack.
2. Determine which of the fractions is largest. Explain to your partner(s) why you believe it is larger.
3. For the other player(s), think of another way to explain why the fraction is larger and share with the group.
4. Pull two more cards and complete steps 2-3 until all cards have been used.

**FORMATIVE ASSESSMENT QUESTIONS**

- Can you think of another way to explain the comparison?
- What equivalent fractions can you identify for \_\_\_\_\_?
- How do you know those fractions are equivalent?
- Which fraction is smaller?
- Which fraction is larger?

## **DIFFERENTIATION**

### **Extension**

- Have students work with fractions greater than 1, such as  $\frac{5}{4}$  and  $\frac{13}{12}$

### **Intervention**

- Allow students to use manipulatives to help compare the fractions.
- Encourage students use number lines to compare the fractions.

Name \_\_\_\_\_ Date \_\_\_\_\_

## More or Less

### Directions

Round 1: Benchmark fraction is  $\frac{3}{4}$

1. Pull a card from the top of the stack.
2. Determine if the fraction is larger or smaller than  $\frac{3}{4}$ . Explain to your partner(s) why you believe it is larger or smaller.
3. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
4. Pull another card and complete steps 2-3 until all cards have been used.

Round 2: Benchmark fraction  $\frac{1}{2}$

1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than  $\frac{1}{2}$ . Explain to your partner(s) why you believe it is larger or smaller.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 3: Benchmark fraction  $\frac{1}{3}$

1. Pull a card from the top of the stack.
2. Identify an equivalent fraction for the benchmark fraction.
3. Determine if the fraction is larger or smaller than  $\frac{1}{3}$ . Explain to your partner(s) why you believe it is larger or smaller.
4. For the other player(s), think of another way to explain why the fraction is more or less and share with the group.
5. Pull another card and complete steps 2-4 until all cards have been used.

Round 4:

1. Pull two cards from the top of the stack.
2. Determine which of the fractions is largest. Explain to your partner(s) why you believe it is larger.
3. For the other player(s), think of another way to explain why the fraction is larger and share with the group.
4. Pull two more cards and complete steps 2-3 until all cards have been used.

$\frac{6}{7}$	$\frac{3}{4}$	$\frac{5}{8}$
$\frac{7}{10}$	$\frac{3}{5}$	$\frac{5}{12}$
$\frac{5}{6}$	$\frac{3}{7}$	$\frac{1}{2}$
$\frac{3}{6}$	$\frac{1}{4}$	$\frac{1}{3}$

$\frac{1}{5}$	$\frac{1}{8}$	$\frac{7}{9}$
$\frac{2}{4}$	$\frac{2}{6}$	$\frac{3}{8}$
$\frac{4}{5}$	$\frac{1}{1}$	$\frac{4}{4}$

## **Practice Task: Closest to 0, $\frac{1}{2}$ , or 1**

*Inspired by Math Solutions Publications Teaching Arithmetic: Lessons for Introducing Fractions by Marilyn Burns*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Constructing the idea that fractions are relationships and that the size or amount of the whole matters is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning from 2<sup>nd</sup> grade, with standards which refer to same-sized shares or equal shares. Students should have knowledge of vocabulary terms such as: *numerator and denominator*.

Some common misconceptions, found in *Math Misconceptions*, that children may have include:

- Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes – circles, squares, and rectangles – they begin to think that all shapes are divided similarly.
- Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set

of 6 vehicles. The student may mistakenly confuse the set of cars as  $\frac{2}{4}$  of the unit instead of  $\frac{2}{6}$  or  $\frac{1}{3}$  (Bamberger, Oberdorf, & Schultz-Ferrell, 2010). Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

### **ESSENTIAL QUESTIONS**

- How can equivalent fractions be identified?
- How can fractions be ordered on a number line?

### **MATERIALS**

- Fraction kits constructed from Fraction Kits Task
- Number line
- Two number cubes (dice) per group

### **GROUPING**

small group or partner

### **TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

**Comments:** Students should have constructed their own fraction kit from the activity, *Fraction Kits*. It is important that you allow time for students to determine where each fraction should be placed on the number line. To enhance the understanding in this task, students may need to construct their own number lines from 0 to 1. This would also be an opportunity to review benchmark fractions by determining where they are placed on the number line. Students may need to stop to get fraction strips to check for equivalence. For students who are unable to determine the appropriate measures, ensure they are exposed to strategies discovered by their peers through classroom discussions.

This task was developed to use as a continuation from the *Fraction Kits* task, but this is not a requirement. If students did not construct their own fraction kits, then they should be able to use other manipulatives that will enable them to have visual representations of equivalent fractions.

#### **Task Directions:**

- First player will roll the dice.
- The smaller number will represent the numerator while the larger will be the denominator.
- Students will discuss if the fractions are closer 0,  $\frac{1}{2}$ , or 1.
- Record the discussion result on the number line with a partner or group members. (Example: Write on the number line where  $\frac{3}{4}$  would be located).
- Continue taking turns until each person has had a 10<sup>th</sup> turn.

- Once all turns have been taken, students should be able to accurately answer assessment questions.

### **FORMATIVE ASSESSMENT QUESTIONS**

- Which fractions are close to  $\frac{1}{2}$ ?
- According to the fraction kit, which fraction is exactly the same as  $\frac{2}{4}$ ?
- How does the relationship between the denominator and the numerator change when it gets closer to 0,  $\frac{1}{2}$ , or 1?
- How does  $\frac{1}{2}$  and 1 compare?
- What are some examples of equivalent fractions you identified during the game?

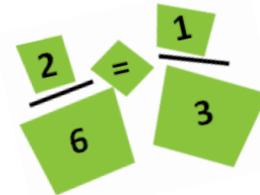
### **DIFFERENTIATION**

#### **Extension**

- Ask students to roll the dice twice for each turn. When they are rolled the first time, they should add the two dice, record it. They should roll them again and record the addition of the two dice again. The smaller number goes on the top and the larger number goes on the bottom. This allows for fractions with larger numerators and denominators (i.e.  $\frac{9}{12}$ ).

#### **Intervention**

- Have the students use the whole piece from the fraction kit as the number line. When they roll their fraction, for example  $\frac{3}{4}$ , they should collect the pieces from the fraction kit, line them up under the whole unit, and then record their fraction using this as a marker. This may have to be modeled before they work independently.



## **Practice Task:** Equivalent Fractions

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF.1** Explain why a fraction  $a/b$  is equivalent to a fraction  $(n \times a)/(n \times b)$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $1/2$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Constructing the idea that fractions are relations and that the size or amount is relative to the whole is a critical step in understanding fractions. Fair sharing contexts also provide learners with opportunities to explore how fractional parts can be equivalent without necessarily being congruent. They may look different but still be the same amount. Students have worked with the concept of fair share or partitioning since 2<sup>nd</sup> grade, with standards which refer to same-sized shares or equal shares. Students should have knowledge of vocabulary terms such as: *numerator* and *denominator*.

Some common misconceptions, found in *Math Misconceptions*, that children have include:

- Dividing nontraditional shapes into thirds, such as triangles, is the same as dividing a rectangle into thirds. If they are only used to dividing traditional shapes – circles, squares, and rectangles – they begin to think that all shapes are divided similarly.
- Children often do not recognize groups of objects as a whole unit. Instead they will incorrectly identify the objects. For example, there may be 2 cars and 4 trucks in a set of 6 vehicles. The student may mistakenly confuse the set of cars as  $2/4$  of the unit instead of  $2/6$  or  $1/3$  (Bamberger, Oberdorf, & Schultz-Ferrell, 2010).

Therefore, it is important that students are exposed to multiple units of measure, various shapes, and denominators other than halves, thirds, and fourths. Additionally, the denominator used as an expression of the whole is a key concept to express for mastery.

### ESSENTIAL QUESTIONS

- What happens to the value of a fraction when the numerator and denominator are multiplied or divided by the same number?
- How are equivalent fractions related?

### MATERIALS

- “Equivalent Fractions,  $\frac{2}{3}$ ,” student recording sheet
- “Equivalent Fractions,  $\frac{3}{4}$ ,” student recording sheet

### GROUPING

individual or partner

### TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

**Comments:** This task allows students to explore the relationship between equivalent fractions and write equations for equivalent fractions using the product and quotient of a fraction equivalent to one.

This task is adapted from Van De Walle, J. (2007). *Elementary and middle school mathematics: Teaching developmentally* (6<sup>th</sup> ed.) Boston: Pearson Education, Inc. See the section on “Equivalent-Fraction Concepts.” (This task is adapted from an activity titled “Slicing Squares” on p. 311.)

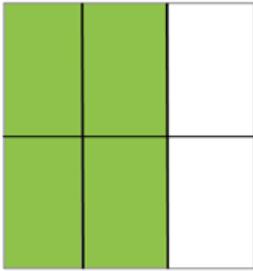
Give students the opportunity to explore equivalent fractions with this task. Encourage students to look for patterns, both in the models as well as in the numerical representations. Equivalent fractions can be thought of as different names for a fraction.

Once students have written equivalent fractions and are able to show that the fraction was multiplied by a fraction equivalent to 1, then begin the discussion about using the inverse operation. Ask students how they can simplify a fraction by dividing it by a fraction equivalent to 1. See the examples below.

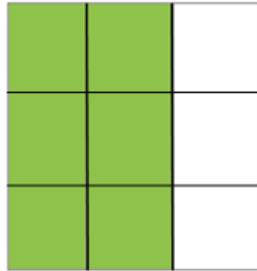
$$\frac{2}{3} = \frac{2}{3} \times \frac{4}{4} = \frac{8}{12}; \frac{8}{12} = \frac{8}{12} \div \frac{4}{4} = \frac{2}{3}$$

$$\frac{3}{4} = \frac{3}{4} \times \frac{2}{2} = \frac{6}{8}; \frac{6}{8} = \frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$$

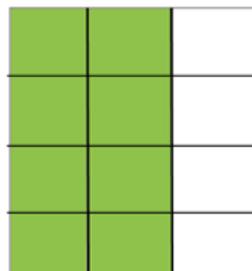
Possible solutions to the “Equivalent Fractions  $\frac{2}{3}$ ,” student recording sheet are shown below:



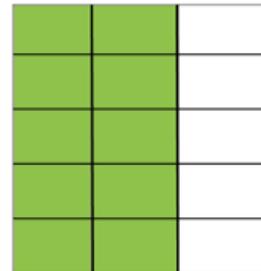
$$\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$$



$$\frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

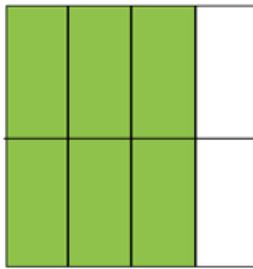


$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$$

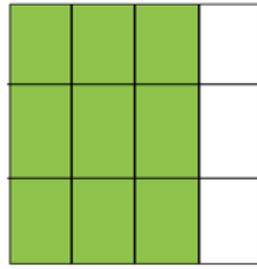


$$\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$$

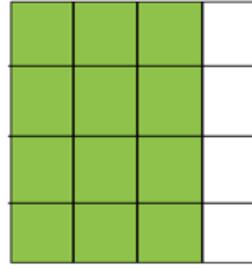
Possible solutions to the “Equivalent Fractions  $\frac{3}{4}$ ,” student recording sheet are shown below.



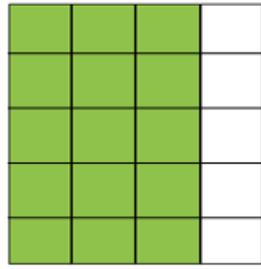
$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$



$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$



$$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$$



$$\frac{3}{4} \times \frac{5}{5} = \frac{15}{20}$$

### Task Directions

Students will follow the directions below from the “Equivalent Fractions  $\frac{2}{3}$ ,” student recording sheet.

Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each model. See the example below.



I cut each piece into equal 3 parts, making 9 pieces.  

$$\frac{2}{3} = \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

Students will follow the directions below from the “Equivalent Fractions  $\frac{3}{4}$ ,” student recording sheet.

Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square.

### FORMATIVE ASSESSMENT QUESTIONS

- Into how many parts did you slice each piece?
- What is a fraction that is equivalent to one? (If the student sliced each piece into three parts, they need to multiply the fraction by  $\frac{3}{3}$ .)
- How could you use equivalent fractions to simplify this fraction (i.e.  $\frac{6}{9}$ )?

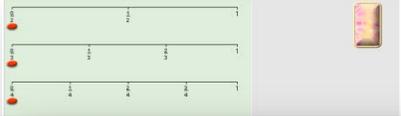
## **DIFFERENTIATION**

### **Extension**

- Invite students to play the Fraction Game.  
<http://illuminations.nctm.org/ActivityDetail.aspx?ID=18>  
 This game requires students to recognize equivalent fractions. Students should think about scenarios from the game that could be presented to the class so that students can discuss strategies and choices available to them.

### **Intervention**

- Some students may benefit from having a table of equivalent fractions. Ask students to complete the table by multiplying the numerator and denominator by the same number. Allow students to refer to this table when working with adding and subtracting fractions with unlike denominators.



Name \_\_\_\_\_ Date \_\_\_\_\_

**Equivalent Fractions**

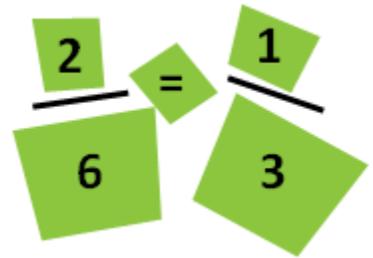
$\frac{2}{6} = \frac{1}{3}$

Find fractions equivalent to the fractions in the table below. Record the equivalent fractions in the white boxes.

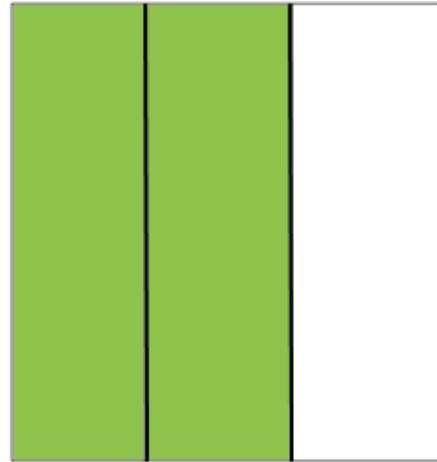
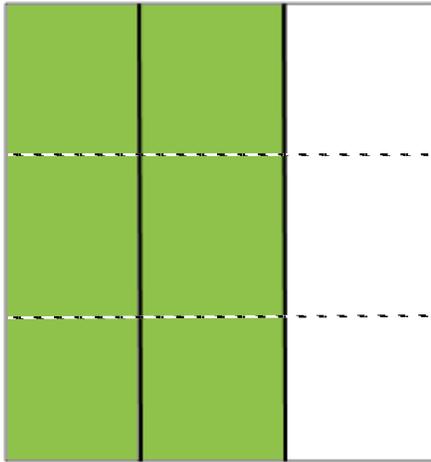
1/2	1/3	1/4	1/5	1/6	1/7	1/8	1/9	1/10	1/11	1/12	1/13	1/14	1/15	1/16	1/17	1/18	1/19	1/20
2/2	2/3	2/4	2/5	2/6	2/7	2/8	2/9	2/10	2/11	2/12	2/13	2/14	2/15	2/16	2/17	2/18	2/19	2/20
3/2	3/3	3/4	3/5	3/6	3/7	3/8	3/9	3/10	3/11	3/12	3/13	3/14	3/15	3/16	3/17	3/18	3/19	3/20
4/2	4/3	4/4	4/5	4/6	4/7	4/8	4/9	4/10	4/11	4/12	4/13	4/14	4/15	4/16	4/17	4/18	4/19	4/20
5/2	5/3	5/4	5/5	5/6	5/7	5/8	5/9	5/10	5/11	5/12	5/13	5/14	5/15	5/16	5/17	5/18	5/19	5/20
6/2	6/3	6/4	6/5	6/6	6/7	6/8	6/9	6/10	6/11	6/12	6/13	6/14	6/15	6/16	6/17	6/18	6/19	6/20
7/2	7/3	7/4	7/5	7/6	7/7	7/8	7/9	7/10	7/11	7/12	7/13	7/14	7/15	7/16	7/17	7/18	7/19	7/20
8/2	8/3	8/4	8/5	8/6	8/7	8/8	8/9	8/10	8/11	8/12	8/13	8/14	8/15	8/16	8/17	8/18	8/19	8/20
9/2	9/3	9/4	9/5	9/6	9/7	9/8	9/9	9/10	9/11	9/12	9/13	9/14	9/15	9/16	9/17	9/18	9/19	9/20
10/2	10/3	10/4	10/5	10/6	10/7	10/8	10/9	10/10	10/11	10/12	10/13	10/14	10/15	10/16	10/17	10/18	10/19	10/20
11/2	11/3	11/4	11/5	11/6	11/7	11/8	11/9	11/10	11/11	11/12	11/13	11/14	11/15	11/16	11/17	11/18	11/19	11/20
12/2	12/3	12/4	12/5	12/6	12/7	12/8	12/9	12/10	12/11	12/12	12/13	12/14	12/15	12/16	12/17	12/18	12/19	12/20
13/2	13/3	13/4	13/5	13/6	13/7	13/8	13/9	13/10	13/11	13/12	13/13	13/14	13/15	13/16	13/17	13/18	13/19	13/20
14/2	14/3	14/4	14/5	14/6	14/7	14/8	14/9	14/10	14/11	14/12	14/13	14/14	14/15	14/16	14/17	14/18	14/19	14/20
15/2	15/3	15/4	15/5	15/6	15/7	15/8	15/9	15/10	15/11	15/12	15/13	15/14	15/15	15/16	15/17	15/18	15/19	15/20
16/2	16/3	16/4	16/5	16/6	16/7	16/8	16/9	16/10	16/11	16/12	16/13	16/14	16/15	16/16	16/17	16/18	16/19	16/20
17/2	17/3	17/4	17/5	17/6	17/7	17/8	17/9	17/10	17/11	17/12	17/13	17/14	17/15	17/16	17/17	17/18	17/19	17/20
18/2	18/3	18/4	18/5	18/6	18/7	18/8	18/9	18/10	18/11	18/12	18/13	18/14	18/15	18/16	18/17	18/18	18/19	18/20
19/2	19/3	19/4	19/5	19/6	19/7	19/8	19/9	19/10	19/11	19/12	19/13	19/14	19/15	19/16	19/17	19/18	19/19	19/20
20/2	20/3	20/4	20/5	20/6	20/7	20/8	20/9	20/10	20/11	20/12	20/13	20/14	20/15	20/16	20/17	20/18	20/19	20/20

Name \_\_\_\_\_ Date \_\_\_\_\_

## Equivalent Fractions – $\frac{2}{3}$

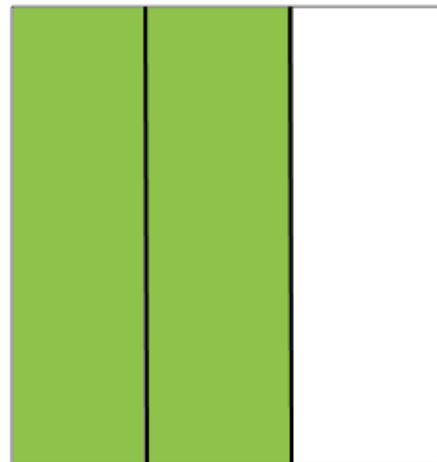
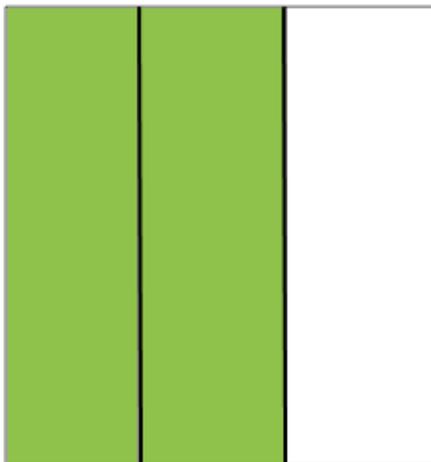


Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square. See the example below.



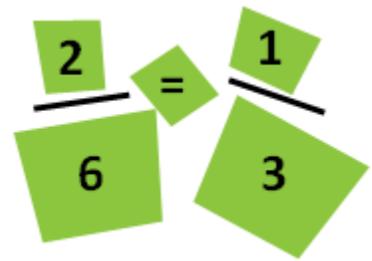
I cut each piece into equal 3 parts,  
making 9 pieces.

$$\frac{2}{3} = \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

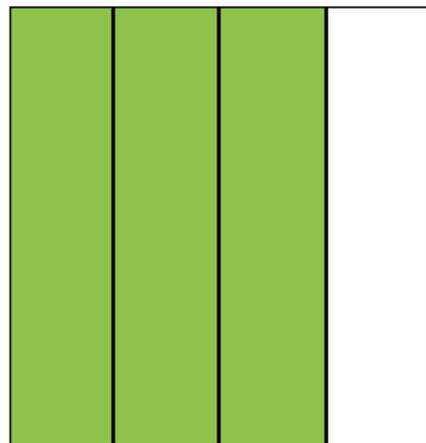
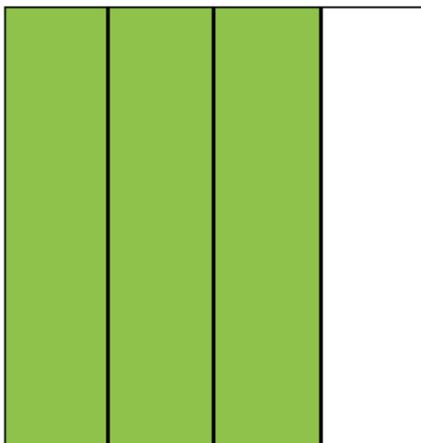
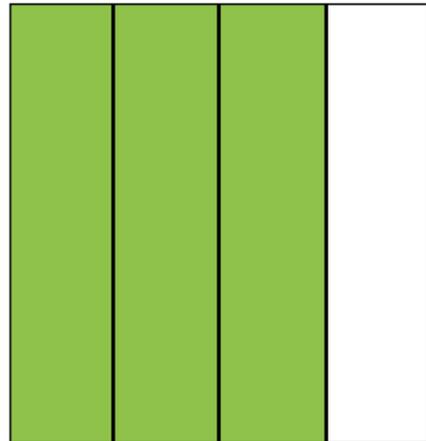
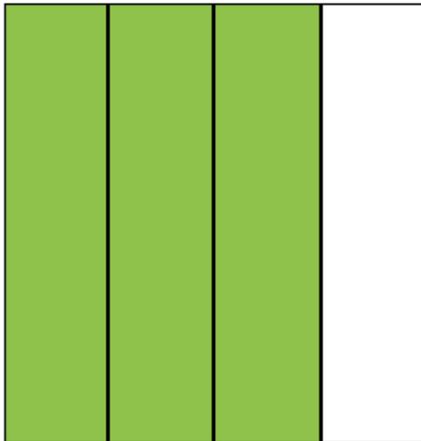


Name \_\_\_\_\_ Date \_\_\_\_\_

## Equivalent Fractions – $\frac{3}{4}$

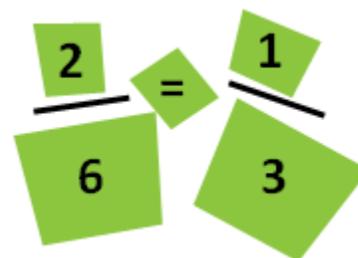


Find fractions that are equivalent to the fraction shown in each square below. Slice the squares by drawing horizontal line segments in each square to create a different but equivalent fraction. Then write an equation for each square.



Name \_\_\_\_\_ Date \_\_\_\_\_

## Equivalent Fractions



Find fractions equivalent to the fractions in the table below. Record the equivalent fractions in the white boxes.

$\frac{1}{2}$						$\frac{3}{4}$		
$\frac{1}{3}$						$\frac{2}{5}$		
$\frac{1}{4}$						$\frac{3}{5}$		
$\frac{1}{5}$						$\frac{4}{5}$		
$\frac{1}{6}$	$\frac{2}{12}$					$\frac{4}{6}$		
$\frac{2}{3}$						$\frac{5}{6}$		

## **Practice Task: Making Fractions**

*Adapted from Actions on Fractions, Navigating through Number and Operations in Grades 3-5.*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Students were familiarized with benchmark fractions in 2<sup>nd</sup> grade. They worked to partition circles and rectangles into halves and thirds. In 3<sup>rd</sup> grade students generated simple equivalent fractions and were required to explain why the fractions were equivalent.

### **ESSENTIAL QUESTIONS**

- How can you compare and order fractions?
- How are benchmark fractions helpful when comparing fractions?
- How do I compare fractions with unlike denominators?

### **MATERIALS**

- Playing cards with all face cards removed

### **GROUPING**

partner

### **TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

**Comments:** Students should follow the rules discussed on the “Making Fractions” student sheet.

How to Play:

- A player deals four cards to themselves and four to their partner.
- Both players make the largest fraction that they can by choosing a numerator and a denominator from the numbers on their cards.
- Compare your fractions. The one with the larger fraction wins all the cards and becomes the new dealer for the next round.
- If the fractions are equal, the round ends in a tie, and the players keep their cards.
- The game continues in this manner until the players have used all cards.
- Each player counts their cards and the one with more cards wins.

### **FORMATIVE ASSESSMENT QUESTIONS**

- Can you think of another way to explain the comparison?
- What equivalent fractions can you identify for \_\_\_\_\_?
- How do you know those fractions are equivalent?
- Which fraction is smaller?
- Which fraction is larger?

### **DIFFERENTIATION**

#### **Extension**

- Change the rules of the game by having players create the smaller fraction.
- Increase the group size to require students to compare and order more than two fractions.
- Instruct students to only use proper fractions.

#### **Intervention**

- Allow students to create their fractions using manipulatives.
- Encourage students to use benchmark fractions to help compare created fractions.
- Encourage students to compare fractions created on number lines.

Name \_\_\_\_\_ Date \_\_\_\_\_

## Making Fractions

### Directions

How to Play:

- A player deals four cards to themselves and four to their partner.
- Both players make the largest fraction that they can by choosing a numerator and a denominator from the numbers on their cards.
- Compare your fractions. The one with the larger fraction wins all the cards and becomes the new dealer for the next round.
- If the fractions are equal, the round ends in a tie, and the players keep their cards.
- The game continues in this manner until the players have used all cards.
- Each player counts their cards and the one with more cards wins.

## **Constructing Task: Factor Findings**

*Adapted from Illuminations provided by the National Council of Teachers of Mathematics*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Students should have developed fluency with multiplying and dividing whole numbers as it was a critical area of focus for grade three. To assess prior knowledge, students will review the multiplication facts for which one factor is 5 or less. A common misconception when listing multiples of numbers is students not listing the number itself. Be sure to emphasize that the smallest multiple is the number itself. Another misconception that may be realized while factoring is that some students think that larger numbers have more factors. As students share factor pairs with partners, this misconception should begin to clear up.

### **ESSENTIAL QUESTIONS**

- What is factor?
- What does it mean to factor?
- What are some situations which factoring is appropriate?

### **MATERIALS**

- color tiles
- graph paper
- crayons
- poster board
- scissors
- glue

## GROUPING

partner

## TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

**Comments:** The students' factor poster will be used to assess the students' understanding and ability to identify the factors of a given number using color tiles, graph paper and factor rainbows.

Throughout the lesson, circulate, observe and question the students as they create their Factor poster. Put the number 24 on the board and have the students find the factors using one of the three methods and record them on a half sheet of paper at the end of the lesson. If you are going to use the number 24, try not to give a group 24 tiles during the lesson. Give students completed factor rainbows. Ask them to create the corresponding arrays and the original number.

In this lesson, students first create factor posters for a variety of different numbers that will be displayed in the classroom to be utilized as a resource throughout the school year. They make discoveries about factors using color tiles, represent their discoveries using graph paper, and display their information on poster board as find factors of an assigned number.

### **Task Directions:**

Begin the lesson by dividing the students into pairs. For this lesson it would work best to pair up students based on ability. Those students of similar abilities should be paired together. Give each pair of students 12 plastic color tiles. Instruct those students to arrange these 12 tiles into an array. Students will create one of the following:



Ask the students to identify how they would express these arrays in number sentences. (6X2, 3X4, 12X1) Some students will have 6X2 and others will have 2X6. This is a great opportunity to discuss the commutative property. Bring attention to the fact that the arrays still look the same they are just positioned differently. Both problems still only provide the factors 6 and 2.

Then write just the numbers on the board: 1,2,3,4,6,12 Explain that these are the factors of 12 because they are the only numbers that can divide evenly with no remainders into that number. This is easily seen when you refer back to the arrays that they created.



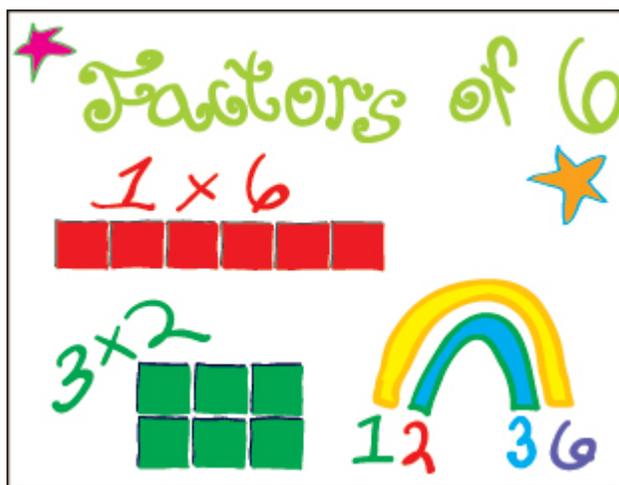
A factor rainbow is a way of showing factor pairs in a list of all of the factors of a number. Factor rainbows are used to check whether a list of factors is correct. To create a factor rainbow, the student must list

the factors in order from least to greatest. They can then draw an arch that links the factor pairs. For square numbers, there will be no connecting arch in the middle; therefore the student can put a square around that number.

Provide the students with another example of finding factors of a number using the color tiles and then creating a factor rainbow. Possible numbers to use would be 16 or 9. Lead the students through the process for this example. Then have the pair of students decide on a number that they would like to find factors of. You may limit this number to 50, depending on the amount of tiles you have available. When they have decided on their number, they can get that many tiles from a bucket of tiles located somewhere in the classroom and use them to find the factors of that number. Instruct the students to create the factor rainbow in their notebooks/journals. Check their factor rainbows to validate the students' understanding before you introduce the main activity.

For the main activity, instruct the students that they are each going to get a number and it is up to them to find all of the factors for that number and create a poster that the entire class can use throughout the year. Give each pair of students a bag of a different amount of color tiles. (ex. 18, 20, 24, 36, 40, 56, 60) Give the bags with the lower amounts to the pairs of students who may be struggling. Instruct the students what to do with tiles using the following directions:

1. Challenge the pairs of students to find as many different arrays as they can, using the color tiles they have been given. They must use all of the tiles each time.
2. Each time they find an array they can then represent it on graph paper.
3. The students color one square on the graph paper for every one plastic color tile in their array. This will form an array on their graph paper.
4. They will cut it out and glue it to their poster and label it with the corresponding factors.
5. They continue this process until they believe that they have found all of the factors.
6. The students then check their factors by creating a factor rainbow at the bottom of their poster.
7. The students must also include a title on their poster. Their poster might look similar to the following:



To conclude the lesson the students will display their posters on the wall and the class will have a gallery walk. During a gallery walk each student will walk around the room and look at everyone's work as if they were in a gallery. They will each be given some post-it notes they can use to anonymously comment on any piece of work and place on the poster. They will also write down two facts they discover after reviewing all of the posters. For example: Just because a number is larger doesn't mean that it has more factors than a smaller number. All even numbers have a factor of two. The number 16 is a perfect square, etc.

### **FORMATIVE ASSESSMENT QUESTIONS**

- What do you notice about the posters that have the number two listed as a factor?
- What would a factor poster of the number three look like?
- What is a number called when the only factors of that number are one and itself?
- When is it useful to know what the factors of a number are?

### **DIFFERENTIATION**

#### **Extension**

- As an extension to this lesson the students can compare posters in order to find the greatest common factors and the least common factors of the numbers on the posters they are comparing.
- Students can play the [Factor Game](#) on the computer.

#### **Intervention**

- Give bags with smaller amounts of factors.

## **Practice Task: Finding Products**

*Adapted from Illuminations provided by the National Council of Teachers of Mathematics*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

**MCC4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Students should have developed fluency with multiplying and dividing whole numbers as it was a critical area of focus for grade three. To assess prior knowledge, students will review the multiplication facts for which one factor is 5 or less. A common misconception when listing multiples of numbers is students not listing the number itself. Be sure to emphasize that the smallest multiple is the number itself. Another misconception that may be realized while factoring is that some students think that larger numbers have more factors. As students share factor pairs with partners, this misconceptions should begin to clear up.

### **ESSENTIAL QUESTIONS**

- What is factor?
- What does it mean to factor?
- What are some situations which factoring is appropriate?

### **MATERIALS**

- Crayons
- Paper

- File Cards

## **GROUPING**

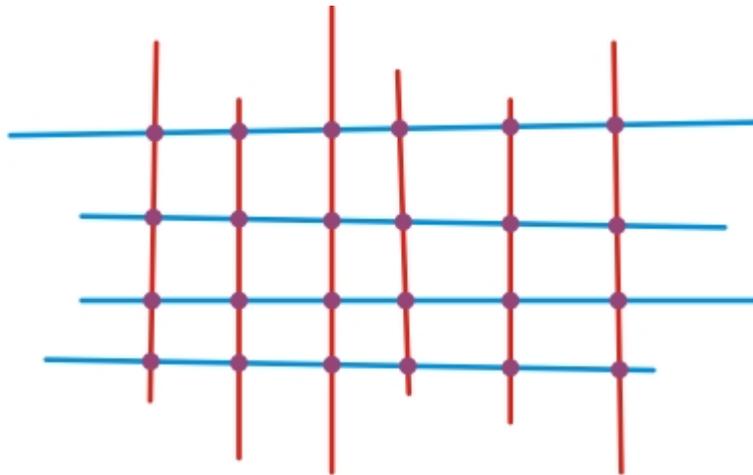
individual

## **TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

Students examine the role of the multiplicative identity, play a multiplication game, and explore products where one of the factors is 6. They also create a "My Personal Multiplication Chart" to record products.

### **Task Directions:**

Provide students with paper and crayons and ask them to draw six blue vertical lines on the paper. Now ask them to draw four red horizontal lines intersecting the vertical lines. Ask them to circle in purple each place there is an intersection and count the number of intersections. Challenge them to identify what multiplication fact they have just demonstrated. Tell them that in this model, the number of rows is given first. [ $4 \times 6 = 24$ .] Ask them to turn their papers a quarter turn and name the multiplication fact now modeled. [ $6 \times 4 = 24$ .]



Encourage them to generate other facts where one factor is 6, including  $6 \times 0$  and  $6 \times 1$ .

Repeat with 7 as a factor.

Distribute index cards to each pair and ask each student to make a set of 10 cards numbered 0 to 9, one to a card.

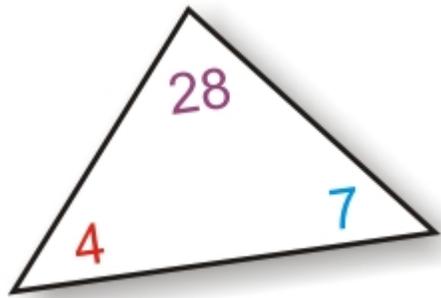
When they have finished, ask them to shuffle the two decks together and stack them face down. Tell them to take turns turning over the top card, multiplying the number drawn by 6 and then saying the product. As each card is used, it should be returned to the bottom of the deck. Give

students time to play, and then ask the class to skip count in unison by 6. Encourage them to do so without looking at the game board. Repeat for 7 as a factor.

Ask students to save the numbered cards for later use.

Next, ask students to make a deck of triangle fact cards for the 6 and 7 tables by putting 2 factors, one of them a 6 or a 7, on 2 of the corners and the product in the third corner. They may wish to use red and blue for the factors and purple for the product.

When they have made triangle fact cards for the facts  $0 \times 6$  to  $9 \times 6$  and  $0 \times 7$  to  $9 \times 7$ , ask each student to cover the product on one card with his or her thumb, show the card to the other student, and ask him or her to tell the product. Encourage the students to separate the cards with the facts they know from those they are less certain.



### **FORMATIVE ASSESSMENT QUESTIONS**

- If you spin a 5, what would the product of 6 and that number be? What multiplication fact would show that?
- If you spin a 1, what would the product of that number and 6 be? Why are you sure of that?
- How could you model with streets and roads the multiplication facts  $3 \times 6 = 18$  and  $6 \times 3 = 18$ ? What is alike between these multiplication sentences? What is different?
- What numbers do you say when you skip count by 7's to 70? Which of these are even numbers?

### **DIFFERENTIATION**

#### **Extension**

- What products can you get when you multiply by 7? How many are even? Will you get an even product when you multiply by 3? By 6? By 8? By 9? How can you tell if the product will be even?
- How many ways can you have a product of 6? Of 12? Of 25? Of 42? Of 1?

### **Intervention**

- Students who need additional practice may use the [Times Table](#) tool.

## **Practice Task: Factor Trail Game**

*Adapted from Illuminations provided by the National Council of Teachers of Mathematics*

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Students should have developed fluency with multiplying and dividing whole numbers as it was a critical area of focus for grade three. To assess prior knowledge, students will review the multiplication facts for which one factor is 5 or less. A common misconception when listing multiples of numbers is students not listing the number itself. Be sure to emphasize that the smallest multiple is the number itself. Another misconception that may be realized while factoring is that some students think that larger numbers have more factors. As students share factor pairs with partners, this misconceptions should begin to clear up.

### **ESSENTIAL QUESTIONS**

- What is factor?
- What does it mean to factor?
- What are some situations which factoring is appropriate?

### **MATERIALS**

- Factor Trail Game
- Calculators (optional)
- Gameboard
- Score Sheet (for each player)
- Dice

## **GROUPING**

partner or small group

## **TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

### **Task Directions**

When students play the Factor Trail game, they have to identify the factors of a number to earn points. Built into this game is cooperative learning — students check one another's work before points are awarded. The score sheet used for this game provides a built-in assessment tool that teachers can use to check their students' understanding.

Explain to students that they will be playing a game involving factoring. Ask them what it means to factor a number, and then ask them to help you find all the factors of a number. You may want to choose a number with a lot of factors (24, 36, 60), or you can roll two dice or spin two spinners to generate the digits of a number randomly. However you choose a number, use the think-pair-share strategy to allow the class to identify the factors: first, give students one minute to "think" individually and come up with some factors of the number; then, give them another minute to discuss their lists with a partner; and finally, record the entire list of factors on the board or overhead projector via a class discussion.

You may then wish to give students a few more numbers to practice on their own before playing the game. Use these additional warm-up problems to determine how well students are able to factor numbers. Then, when students begin to play the game for practice, spend additional time with students who had difficulty.

After the warm-up, distribute the Factor Trail Game to all students. Note that two students can share the game board and rules that appear on the first and second pages, but all students will need their own score sheet.

Players move around the game board, landing on numbered squares. When landing on a square with a number, students should list all of the factors of that number on their score sheet. When a student believes that she has listed all of the factors, her opponent checks the list. If her opponent identifies any factors not on the player's list, or if the opponent identifies any number on the player's list that is not a factor of the number, the opponent receives 10 points for identifying the error. (If the opponent notices multiple errors, 10 points are earned for each error.) If the player made no errors, however, then she receives points for that turn equal to the sum of the factors of the number.

*Example:* A player lands on 18. On her score sheet, she lists 1, 2, 3, 4, 6, 9, and 18 as factors. Upon indicating that she has completed her list, her opponent points out that 4 is not a factor of 18. Consequently, the opponent receives 10 points for identifying the error. On the other hand, if she had not included 4 on her list, she would have correctly identified all of the factors and received  $1 + 2 + 3 + 6 + 9 + 18 = 39$  points.

The game can be played with or without calculators. The use of calculators does not greatly influence the game, as students must still understand the concept and skill of factoring to be successful. However, if a secondary objective of the lesson is to have students practice mental arithmetic, then calculators should not be used.

As students play the game, circulate to offer assistance where necessary. This may involve settling a dispute between two students, or it may require intervening when you notice that students are making mistakes not caught by their opponents.

With a few minutes left in class, you may wish to pause all games and conduct a brief discussion using the questions that appear in the Formative Assessment Questions section below.

### **FORMATIVE ASSESSMENT QUESTIONS**

- Which number on the game board has the most factors?
- For which number on the trail will a player earn the most points?
- In general, how many points are earned for a prime number?
- Collect the score sheets of all students. The score sheets can be used to determine if students were correctly finding the factors of numbers. One of the benefits of using the score sheet occurs when a 0 is entered in the "Points Earned" column. This indicates that the student made a mistake when finding the factors of that number, so it is easier to identify areas of difficulty.

### **DIFFERENTIATION**

#### **Extension**

- Change the numbers on the game board. Note that all of the numbers are less than 100. For a more advanced game, include numbers in the hundreds or thousands.

#### **Intervention**

- Change the numbers on the game board. The numbers could change to being numbers below 50. The numbers will need to be frequently repeated, but the repetition will help students to develop the concept of factoring prior to working with larger numbers.

### **TECHNOLOGY CONNECTION**

<http://illuminations.nctm.org/LessonDetail.aspx?ID=L719>

This is an interactive activity that enables students to enhance their understanding of factorization.

## **Practice Task:** The Sieve of Eratosthenes

### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Many students may not understand why “one” is not a prime number. Be sure to allow for plenty of discussion about this. One is neither a prime nor a composite number. A prime number is one with exactly two positive, unique divisors, itself and one. One has only one positive, unique divisor. It cannot be written as a product of two factors, neither of which is the number itself, so one is also not composite. It falls in a class of numbers called units. These are the numbers whose reciprocals are also whole numbers. For more information go to:

[http://mathforum.org/dr.math/faq/faq\\_prime\\_num.html](http://mathforum.org/dr.math/faq/faq_prime_num.html)

Students are asked to write an expression for given multiples in this task. Some may not be sure what the question is asking, providing a prime opportunity for discussion. Because all even numbers are multiples of two, the algebraic expression for multiples of two is  $2n$ . Similarly, the multiples of 5 can be expressed as  $5n$ .

### **ESSENTIAL QUESTIONS**

- How do we know if a number is prime or composite?
- How can we determine whether a number is odd or even?
- How are factors and multiples defined?

### **MATERIALS**

- “Sieve of Eratosthenes” Recording Sheet
- “Exploring the Sieve of Eratosthenes” Recording Sheet
- Colored pencils, markers, highlighters or crayons

## **GROUPING**

individual or partner

## **TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION**

This task allows students to create a list of prime numbers between 1 and 100.

### **Comments**

Students should use a different color for each new prime number. Using different colors works better, but students might need to use a combination of crayons, highlighters, colored pencils, and markers. A different symbol could be used for each prime number (i.e., \*, #, etc.), but using colors might be more effective. Have students draw only one line through numbers so when they cross out a number more than once, they can do so in a different direction. This will help students find numbers that are multiples of several numbers.

One way to introduce this task is to model the first one or two steps on the overhead. Discuss the patterns students see. Write these on the board or chart paper as students share them. Each student needs to complete his/her own Sieve of Eratosthenes. Let students share their observations. Record these on a chart or the board. The “Exploring the Sieve of Eratosthenes” student recording sheet will help students determine factors and multiples.

### **Task Directions**

Use a 0-100 chart to complete the following:

1. Draw a line through the number 1, because it is not a prime number.
2. Circle the number 2 with yellow, because it is the smallest prime number. Draw a line through every multiple of 2 with yellow. How do you know which numbers to cross out? Write an algebraic expression for the numbers you crossed out with yellow.
3. Circle the number 3 with blue. This is the next prime number. Now, draw a line through every multiple of 3 with blue. What do you notice about the number 6? What do you think it means when a number is crossed out with two colors – in this case yellow and blue?
4. Circle the next open number, 5 with red. Draw a line through all multiples of 5 with red. Write an algebraic expression for the numbers you crossed out in red.
5. Circle the next open number with orange. Draw a line through all multiples of 7 with orange.
6. Continue doing this with different colors until all the numbers through 100 have either been circled or crossed out.
7. Write to explain what you noticed about the circled numbers.

## **FORMATIVE ASSESSMENT QUESTIONS**

- What do all of the multiples of two have in common? Multiples of five?
- If you multiply by two will you always get an even answer (a multiple of two)? Why?
- If you multiply by five will you always get a multiple of five? Why?

## **DIFFERENTIATION**

### **Extension**

- Students can write a letter to an “absent” classmate about how to use the Sieve of Eratosthenes to find factors of numbers.

### **Intervention**

- Have students use a calculator or the computer to generate the multiples of numbers.

Name \_\_\_\_\_ Date \_\_\_\_\_

## Sieve of Eratosthenes

Eratosthenes (275-194 B.C., Greece) devised a 'sieve' to discover prime numbers. A sieve is like a strainer that you drain spaghetti through when it is done cooking. The water drains out, leaving your spaghetti behind. Eratosthenes's sieve drains out composite numbers and leaves prime numbers behind.



Use a hundred chart below to complete the following:

1. Put a square around the number 1, because it is neither prime number nor composite.
2. Circle the number 2 with yellow, because it is the smallest prime number. Draw a line through every multiple of 2 with yellow. How do you know which numbers to cross out? Write an algebraic expression for the numbers you crossed out with yellow.
3. Circle the number 3 with blue. This is the next prime number. Now, draw a line through every multiple of 3 with blue. What do you notice about the number 6? What do you think it means when a number is crossed out with two colors – in this case yellow and blue?
4. Circle the next open number, 5 with red. Draw a line through all multiples of 5 with red. Write an algebraic expression for the numbers you crossed out in red.
5. Circle the next open number with orange. Draw a line through all multiples of 7 with orange.
6. Continue doing this with different colors until all the numbers through 100 have either been circled or crossed out.
7. Write to explain what you noticed about the circled numbers.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Name \_\_\_\_\_ Date \_\_\_\_\_

## Exploring the Sieve of Eratosthenes Recording Sheet



Using your Sieve of Eratosthenes list the prime numbers up to 100.

\_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_,  
 \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_,  
 \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

Complete the table below. Choose a number between 40 and 60 for the last two rows, and then complete the chart using the numbers chosen.

Find this number on your hundreds board:	Prime or Composite?	List the factors of the number
3		
9		
12		
15		
17		
21		
24		
29		

Look at your chart and record 3 observations from your work.

---



---



---

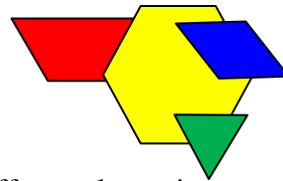


---



---

## **Culminating Task: Pattern Block Puzzles**



### **STANDARDS FOR MATHEMATICAL CONTENT**

**MCC4.NF.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as  $\frac{1}{2}$ . Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

**MCC4.NF.1** Explain why a fraction  $\frac{a}{b}$  is equivalent to a fraction  $\frac{n \times a}{n \times b}$  by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

**MCC4.OA.1** Interpret a multiplication equation as a comparison, e.g., interpret  $35 = 5 \times 7$  as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

**MCC4.OA.4** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite

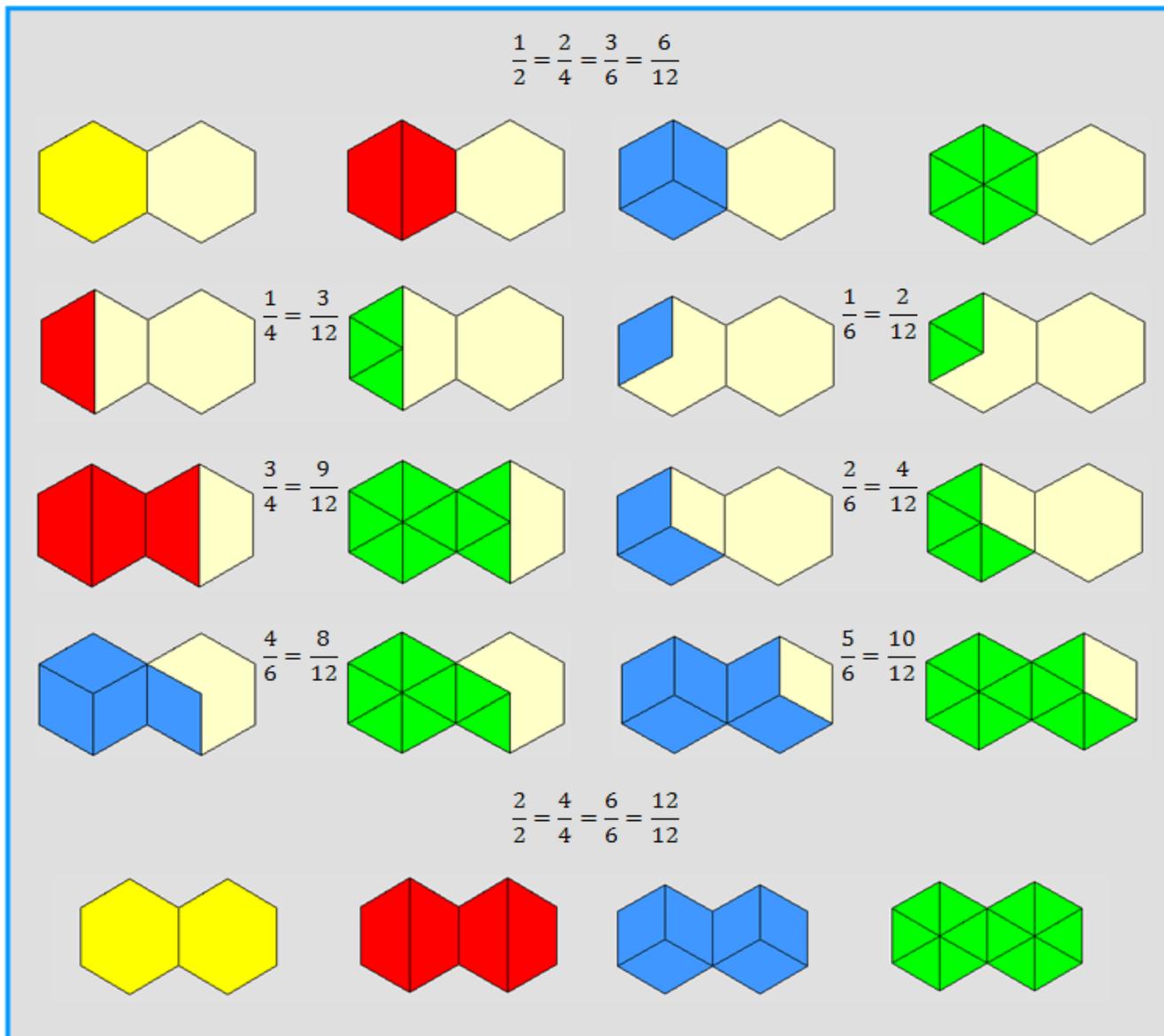
### **STANDARDS FOR MATHEMATICAL PRACTICE**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

### **BACKGROUND KNOWLEDGE**

Encourage students to organize their thinking to be sure they have found as many ways as possible to represent equivalent fractions. Possible equivalent fractions are shown below. Before asking students to work on this task, be sure students are able to:

- identify the number of equal pieces needed to cover one whole as the denominator
- show equivalent fractions with the pattern blocks
- record on the student sheet equivalent fractions (either by coloring or gluing die cut pattern blocks)
- write an equation which shows the equivalent fractions



**ESSENTIAL QUESTIONS**

- How can we find equivalent fractions?
- In what ways can we model equivalent fractions?
- How can we find equivalent fractions and simplify fractions?
- How can identifying factors and multiples of denominators help to identify equivalent fractions?
- How can the denominator tell me if a fraction is in the smallest equivalent fraction?

## MATERIALS

- Pattern blocks
- “Pattern Block Puzzles” student recording sheet
- Die cut paper pattern blocks (or pencils and crayons for recording findings)
- “Pattern Block Puzzles, Version 2” student recording sheet, for intervention students only

## GROUPING

individual or partner

## TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

**Comments:** A limited set of pattern blocks should be available for the students. It may be helpful to create sets of pattern blocks in plastic bags beforehand for ease in distributing them to the student groups. One possible “limited set” of pattern blocks might have 1 hexagon, 1 trapezoid, 1 rhombus, and 3 triangles. This set would require students to apply their understanding of equivalent fractions as developed in this unit.

If available, students can glue die-cut pattern block pieces. Alternately, students can manipulate the virtual pattern blocks online and easily print and then label their work. One site for pattern blocks is: <http://gingerbooth.com/flash/patblocks/patblocks.php>.

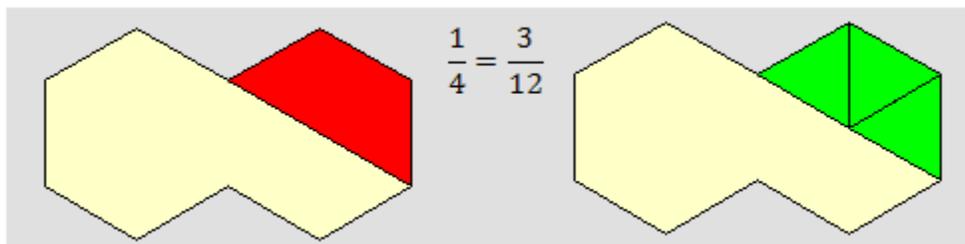
This task could be introduced by reading *The Hershey’s Milk Chocolate Bar Fractions Book*, by Jerry Pallotta, illustrated by Rob Bolster or another story about equivalent fractions.

### **Task Directions:**

Students will follow directions below from the “Pattern Block Puzzle” student recording sheet.

- Obtain a set of pattern blocks.
- Use the pattern blocks to show equivalent fractional amounts.
- Record the equivalent fractions on the “one whole” pairs below.
- Write a number sentence for each equivalent fraction. (See example.)
- How many equivalent fractions can you find?
- Use what you know about factors and multiples to identify two equivalent fractions without using the pattern blocks.
- Which equivalent fractions are the smallest of all of its equivalent fractions? How do you know?

Example:



## FORMATIVE ASSESSMENT QUESTIONS

- How are you keeping your work organized?
- Have you found all of the possible equivalent fractions? How do you know?
- How do you know these two fractions are equivalent?
- How many different illustrations can be created to show equivalent fractions? How do you know?

## DIFFERENTIATION

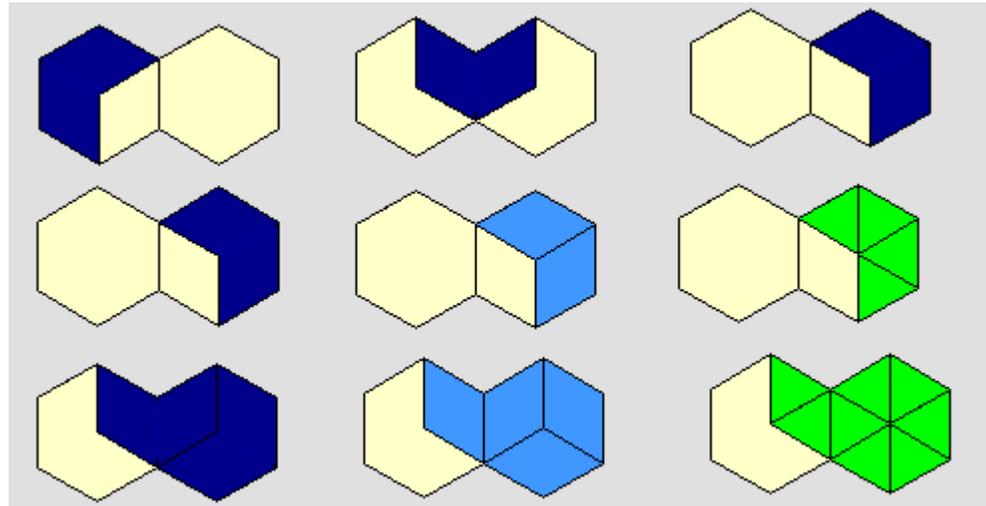
### Extension

- Once students have completed the task above, this lesson can be extended to other shapes or sets that can serve as a template to show equivalent fractions. For example:
  - ~ Use a set of 8, 10 or 12 unit cubes to represent one whole.
  - ~ Use a hexagon and rhombus to represent one whole or use a hexagon and a trapezoid to represent one whole.
- How could you show one-third of this figure? (See diagram to the right.) Students will need to create a “new” pattern block made from two rhombi to show one-third. Students could then be asked to find equivalent fractions for thirds.

Three examples  
of one third

$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$$

$$\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$$



Of course students can also show  $\frac{3}{3}$  one whole.

- Encourage students to add some of the fractions found in this task. This will give students an opportunity to begin exploring improper fractions and mixed numbers. Be sure pattern block models are used to show the addition problems.

### Intervention

- Allow students to record equivalent fractions using pre-cut paper pattern blocks.

- Allow students to use the “Pattern Block Puzzles, Version 2” student recording sheet. In version 2, the whole is represented by the yellow hexagon. Students should be able to find most if not all of the possible equivalent fractions.
- Possible solutions:

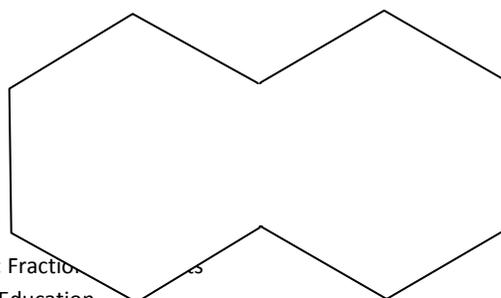
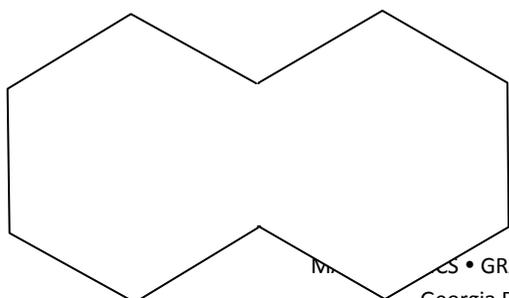
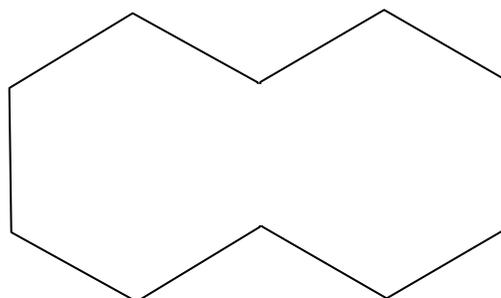
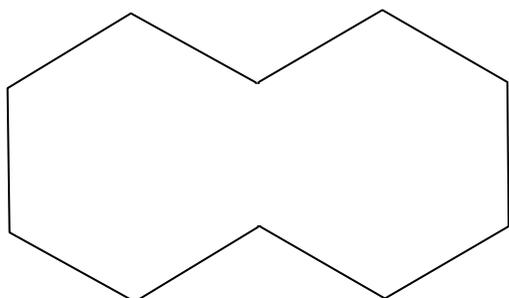
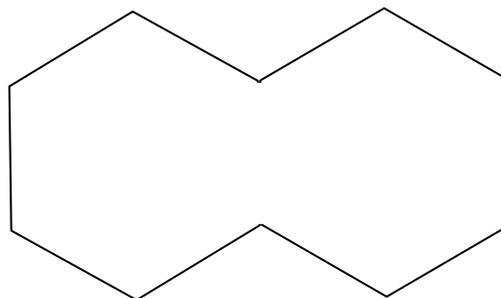
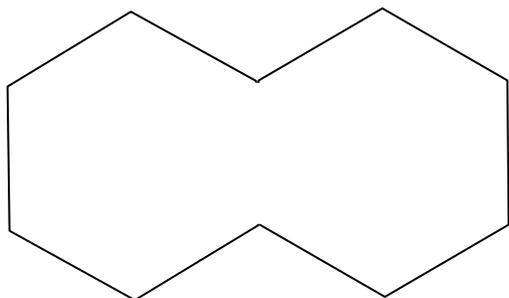
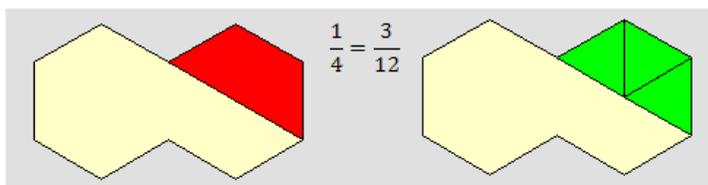
$$\frac{1}{2} = \frac{3}{6}; \frac{1}{3} = \frac{2}{6}; \frac{2}{3} = \frac{4}{6}; 1 = \frac{2}{2}; 1 = \frac{3}{3}; 1 = \frac{6}{6}.$$

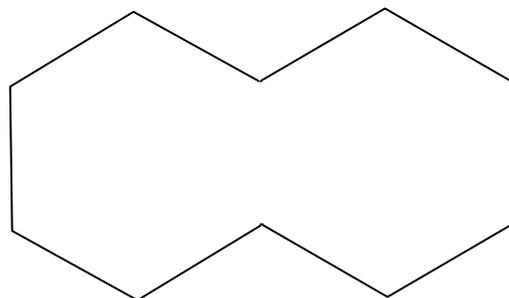
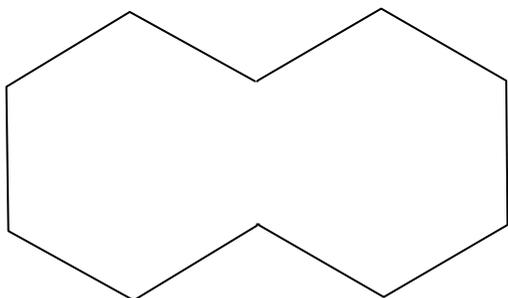
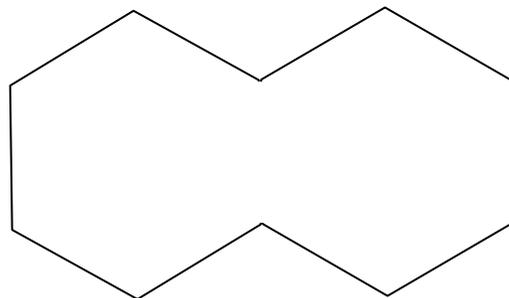
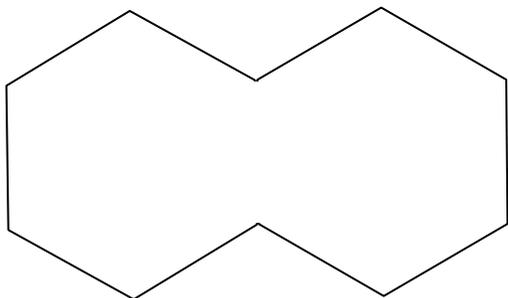
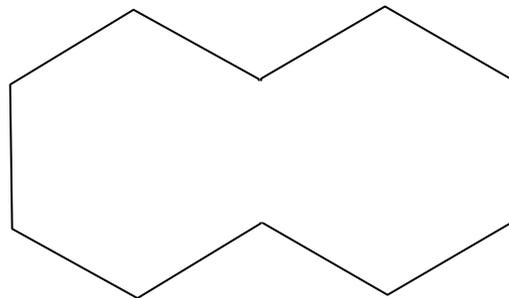
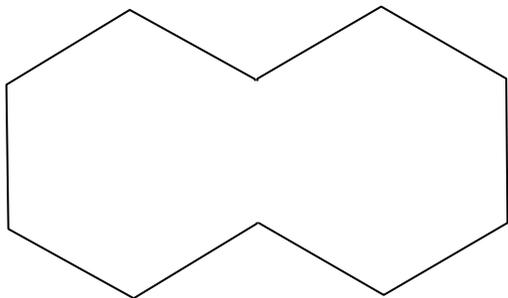
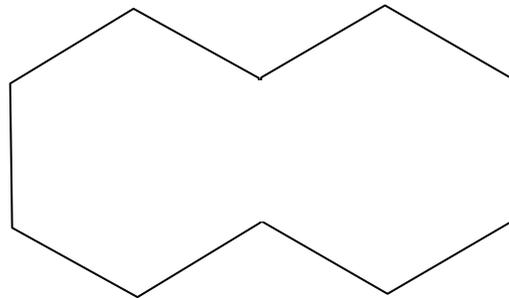
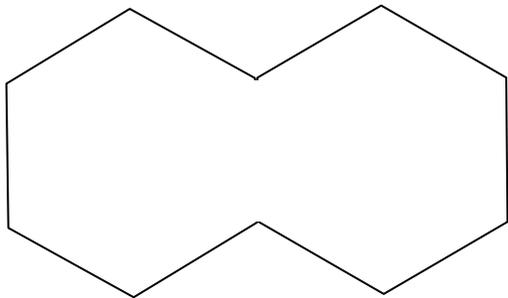
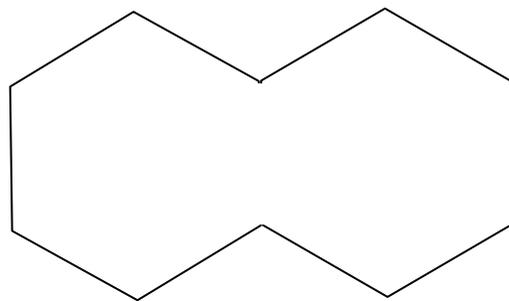
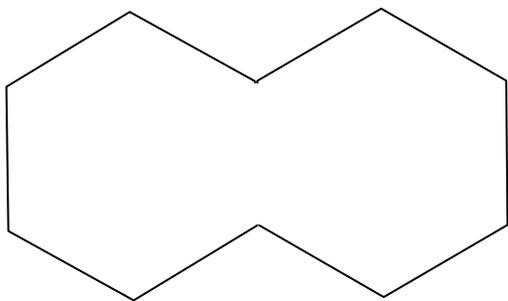
Name \_\_\_\_\_ Date \_\_\_\_\_

## Pattern Block Puzzles

### Directions

- Obtain a set of pattern blocks from your teacher.
- Use the pattern blocks to show equivalent fractional amounts.
- Record the equivalent fractions on the “one whole” pairs below.
- Write a number sentence for each equivalent fraction. (See example.)
- How many equivalent fractions can you find?
  - Use what you know about factors and multiples to identify two equivalent fractions without using the pattern blocks.
  - Which equivalent fractions are the smallest of all of its equivalent fractions? How do you know?

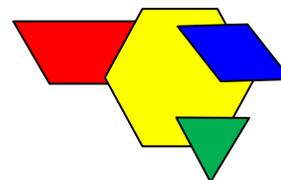




Name \_\_\_\_\_ Date \_\_\_\_\_

## Pattern Block Puzzles

### Version 2



#### Directions

- Obtain a set of pattern blocks.
- Use the pattern blocks to show equivalent fractional amounts.
- Record the equivalent fractions on the “one whole” pairs below.
- Write a number sentence for each equivalent fraction. (See example.)
- How many equivalent fractions can you find?
  - Use what you know about factors and multiples to identify two equivalent fractions without using the pattern blocks.
  - Which equivalent fractions are the smallest of all of its equivalent fractions? How do you know?

$$\frac{3}{6} = \frac{1}{2}$$

